There are two times in a man’s life when he should not speculate:
...when he can’t afford it, and when he can.
—Mark Twain

Semester: Fall 2009

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Lectures: Saturdays from 10:00-13:00 (subject to change)

Recommended Text:

Grading:
Attendance & Homeworks: 60%
Two Midterms: 20% each

Motivation:
However simple, the majority of the financial derivatives pricing problems cannot be exactly solved; accordingly, one needs to consult numerical methods. The state-of-the-art numerical schemes for option pricing problems can be based on either (1) simulation processes (e.g. Monte Carlo), or (2) lattice methods (e.g. Binomial Lattices), or (3) solutions of the partial differential equations (e.g. Finite Difference or Finite Element Methods). Among them, the Finite Difference Method (FDM) has been very fashionable due to its ease of implementation.

Objectives:
In this course, we will use FDM to solve the partial differential equations arising in connection with the European and American option pricing models. Utilization of FDM, in turn, requires the effective use of linear system solvers. Initially, we will thus focus on (1) direct methods (e.g. Gaussian elimination), (2) classical iterative methods (Jacobi, Gauss-Seidel, and SOR methods), (3) projection methods (Steepest Descent and Conjugate Gradient methods), and (4) Krylov subspace methods (e.g. GMRES) for the solution of linear systems of equations. Throughout the course, our primary means of implementation will be MATLAB.

Primary References:
Duffy: *Finite Difference Methods in Financial Engineering: A PDE Approach*
Quarteroni, Sacco & Saleri: *Numerical Mathematics*

Secondary References:
Iserles: *A First Course in the Numerical Analysis of Differential Equations*
Richtmyer & Morton: *Difference Methods for Initial-Value Problems*
Willmot, Howison & Dewynne: *The Mathematics of Financial Derivatives*
Achdou: *Computational Methods for Option Pricing*
Topper: *Financial Engineering with Finite Elements*
Glasserman: *Monte Carlo Methods in Financial Engineering*
Seydel: *Tools for Computational Finance*
Contents (subject to change)

• Principles of Numerical Mathematics
  – Number representation, rounding, and truncation
  – Error propagation, conditioning, and instability
  – Order of convergence and computational complexity

• Rootfinding for Nonlinear Equations
  – The Bisection Method
  – Chord, secant, regula falsi, and Newton’s methods
  – Fixed point iterations for nonlinear equations

• Solving Systems of Linear Equations
  – Vector and matrix norms
  – Condition number for a matrix
  – Direct methods (LU and Cholesky factorizations)
  – Iterative methods for solving systems of linear equations (Jacobi, Gauss-Seidel, SOR)
  – Projection methods for solving systems of linear equations (steepest descent, conjugate gradients)
  – Krylov subspace methods (GMRES)

• Finite Difference Methods for Partial Differential Equations
  – Introduction and classification of PDEs
  – Numerical solution by finite difference methods
  – Explicit methods for the heat equation
  – Implicit methods for the heat equation
  – The Crank-Nicolson method for the heat equation
  – The upwinding method for the transport equation
  – Convergence, consistency, and stability

• Finite Difference Methods for European Options
  – Applying finite difference methods to the Black-Scholes equation
  – Pricing a vanilla European option by an explicit method
  – Pricing a vanilla European option by a fully implicit method
  – Pricing a barrier option by the Crank-Nicolson method

• Finite Difference Methods for American Options
  – The terminal, boundary-value problem
  – The obstacle problem
  – A finite difference method for the obstacle problem
  – The projected SOR method
  – A finite difference method for American options