Seventh Edition

CHAPTER 15

VECTOR MECHANICS FOR ENGINEERS: DYNAMICS

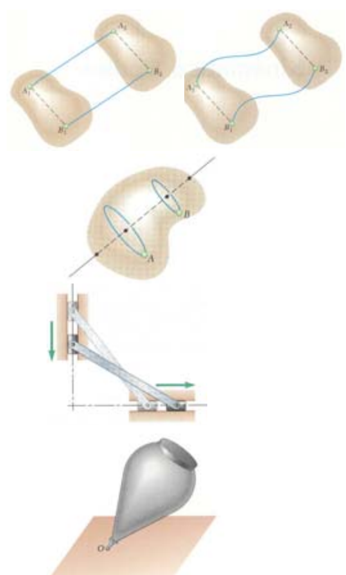
Ferdinand P. Beer
E. Russell Johnston, Jr.

Lecture Notes:
J. Walt Oler
Texas Tech University

Kinematics of Rigid Bodies

Contents

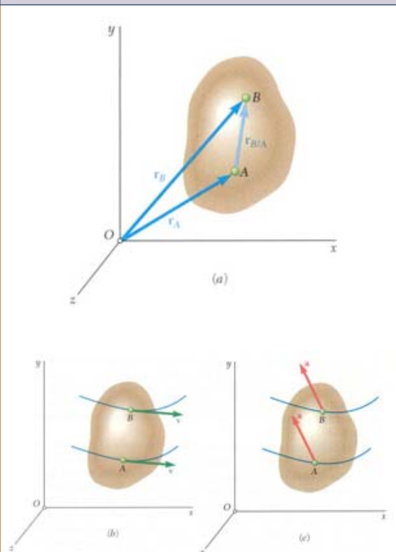
Introduction



- Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.
- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation
  - rotation about a fixed axis
  - general plane motion
  - motion about a fixed point
  - general motion

Seventh Edition
Vector Mechanics for Engineers: Dynamics

Translation



- Consider rigid body in translation:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.
- For any two particles in the body,

  $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$
- Differentiating with respect to time,

  $\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$

  $\vec{v}_B = \vec{v}_A$

  All particles have the same velocity.
- Differentiating with respect to time again,

  $\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$

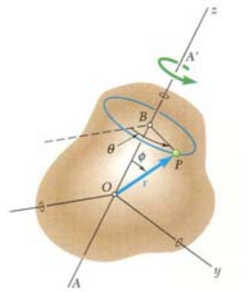
  $\vec{a}_B = \vec{a}_A$

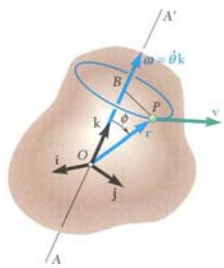
  All particles have the same acceleration.

Seventh Edition

Vector Mechanics for Engineers: Dynamics

Rotation About a Fixed Axis. Velocity





- Consider rotation of rigid body about a fixed axis *AA'*
- Velocity vector $\vec{v} = d\vec{r}/dt$ of the particle *P* is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0} (r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

- The same result is obtained from

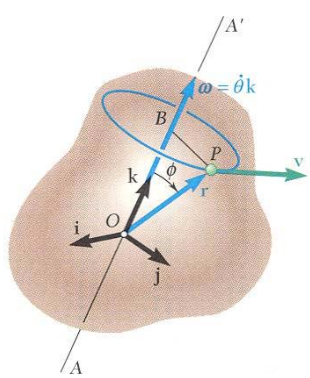
$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = \textit{angular velocity}$$

Seventh Edition

Vector Mechanics for Engineers: Dynamics

Rotation About a Fixed Axis. Acceleration



- Differentiating to determine the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \vec{v}$$

- $\dfrac{d\vec{\omega}}{dt} = \vec{\alpha} = \textit{angular acceleration}$

$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$

- Acceleration of *P* is combination of two vectors,

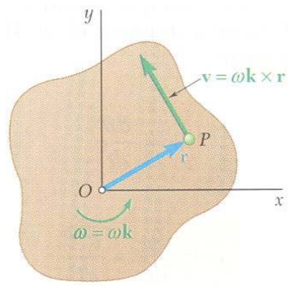
$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

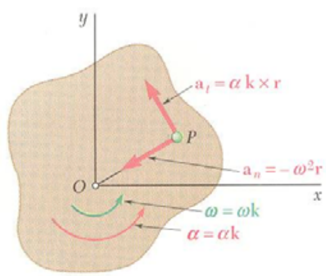
$\vec{\alpha} \times \vec{r} =$ tangential acceleration component

$\vec{\omega} \times (\vec{\omega} \times \vec{r}) =$ radial acceleration component

Vector Mechanics for Engineers: Dynamics

Rotation About a Fixed Axis. Representative Slab





- Consider the motion of a representative slab in a plane perpendicular to the axis of rotation.
- Velocity of any point *P* of the slab,

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega\vec{k} \times \vec{r}$$
$$v = r\omega$$

- Acceleration of any point *P* of the slab,

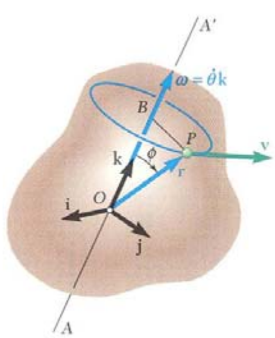
$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$
$$= \alpha\vec{k} \times \vec{r} - \omega^2\vec{r}$$

- Resolving the acceleration into tangential and normal components,

$$\vec{a}_t = \alpha\vec{k} \times \vec{r} \qquad a_t = r\alpha$$
$$\vec{a}_n = -\omega^2\vec{r} \qquad a_n = r\omega^2$$

Vector Mechanics for Engineers: Dynamics

Equations Defining the Rotation of a Rigid Body About a Fixed Axis



- Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.
- Recall $\omega = \frac{d\theta}{dt}$ or $dt = \frac{d\theta}{\omega}$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$

- *Uniform Rotation*, $\alpha = 0$:

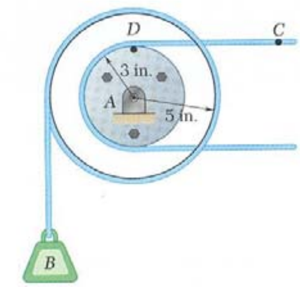
$$\theta = \theta_0 + \omega t$$

- *Uniformly Accelerated Rotation*, $\alpha =$ constant:

$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 5.1



Cable C has a constant acceleration of 9 in/s^2 and an initial velocity of 12 in/s, both directed to the right.

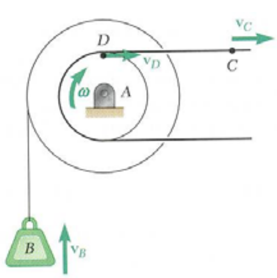
Determine *(a)* the number of revolutions of the pulley in 2 s, *(b)* the velocity and change in position of the load B after 2 s, and *(c)* the acceleration of the point D on the rim of the inner pulley at t = 0.

SOLUTION:

- Due to the action of the cable, the tangential velocity and acceleration of D are equal to the velocity and acceleration of C. Calculate the initial angular velocity and acceleration.
- Apply the relations for uniformly accelerated rotation to determine the velocity and angular position of the pulley after 2 s.
- Evaluate the initial tangential and normal acceleration components of D.

Vector Mechanics for Engineers: Dynamics

Sample Problem 5.1



SOLUTION:

- The tangential velocity and acceleration of D are equal to the velocity and acceleration of C.

$$(\vec{v}_D)_0 = (\vec{v}_C)_0 = 12\,\text{in./s} \rightarrow \qquad (\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$

$$(v_D)_0 = r\omega_0 \qquad (a_D)_t = r\alpha$$

$$\omega_0 = \frac{(v_D)_0}{r} = \frac{12}{3} = 4\,\text{rad/s} \qquad \alpha = \frac{(a_D)_t}{r} = \frac{9}{3} = 3\,\text{rad/s}^2$$

- Apply the relations for uniformly accelerated rotation to determine velocity and angular position of pulley after 2 s.

$$\omega = \omega_0 + \alpha t = 4\,\text{rad/s} + \left(3\,\text{rad/s}^2\right)(2\,\text{s}) = 10\,\text{rad/s}$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (4\,\text{rad/s})(2\,\text{s}) + \tfrac{1}{2}\left(3\,\text{rad/s}^2\right)(2\,\text{s})^2$$

$$= 14\,\text{rad}$$

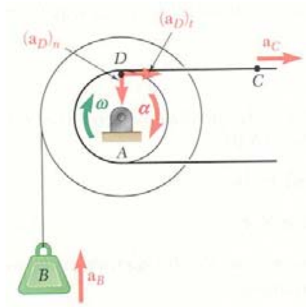
$$N = (14\,\text{rad})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right) = \text{number of revs} \qquad \boxed{N = 2.23\,\text{rev}}$$

$$v_B = r\omega = (5\,\text{in.})(10\,\text{rad/s}) \qquad \boxed{\vec{v}_B = 50\,\text{in./s} \uparrow}$$

$$\Delta y_B = r\theta = (5\,\text{in.})(14\,\text{rad}) \qquad \boxed{\Delta y_B = 70\,\text{in.}}$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 5.1

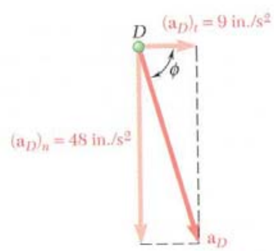


- Evaluate the initial tangential and normal acceleration components of D.

$$(\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$

$$(a_D)_n = r_D\omega_0^2 = (3\,\text{in.})(4\,\text{rad/s})^2 = 48\,\text{in/s}^2$$

$$\boxed{(\vec{a}_D)_t = 9\,\text{in./s}^2 \rightarrow \qquad (\vec{a}_D)_n = 48\,\text{in./s}^2 \downarrow}$$



Magnitude and direction of the total acceleration,

$$a_D = \sqrt{(a_D)_t^2 + (a_D)_n^2} = \sqrt{9^2 + 48^2}$$

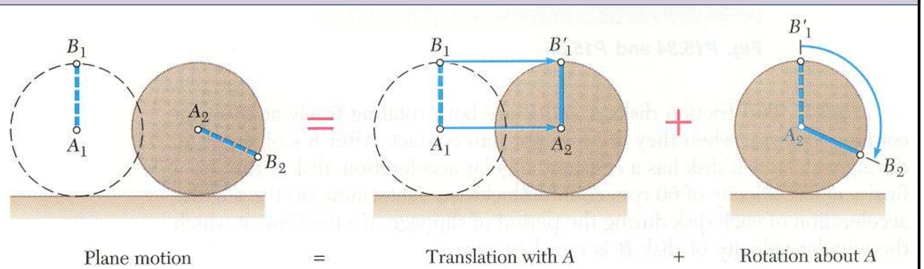
$$\boxed{a_D = 48.8\,\text{in./s}^2}$$

$$\tan\phi = \frac{(a_D)_n}{(a_D)_t} = \frac{48}{9}$$

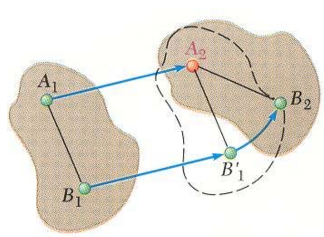
$$\boxed{\phi = 79.4^\circ}$$

Vector Mechanics for Engineers: Dynamics

General Plane Motion



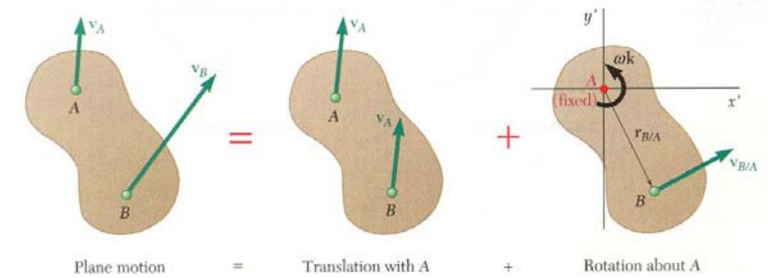
- *General plane motion* is neither a translation nor a rotation.
- General plane motion can be considered as the *sum* of a translation and rotation.
- Displacement of particles A and B to A_2 and B_2 can be divided into two parts:
  - translation to A_2 and B_1'
  - rotation of B_1' about A_2 to B_2

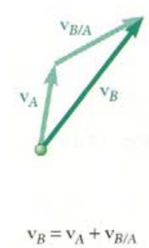


Seventh Edition

Vector Mechanics for Engineers: Dynamics

Absolute and Relative Velocity in Plane Motion





- Any plane motion can be replaced by a translation of an arbitrary reference point *A* and a simultaneous rotation about *A*.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

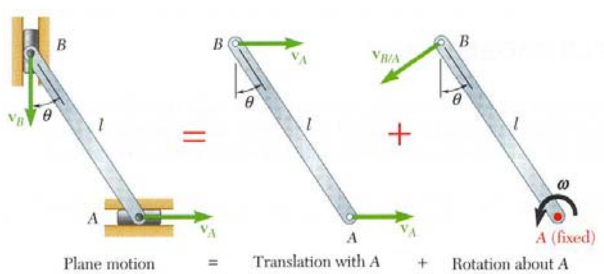
$$\vec{v}_{B/A} = \omega\vec{k}\times\vec{r}_{B/A} \qquad v_{B/A} = r\omega$$

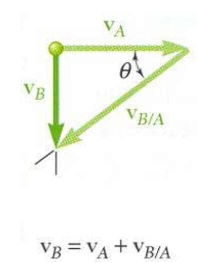
$$\vec{v}_B = \vec{v}_A + \omega\vec{k}\times\vec{r}_{B/A}$$

Seventh Edition

Vector Mechanics for Engineers: Dynamics

Absolute and Relative Velocity in Plane Motion





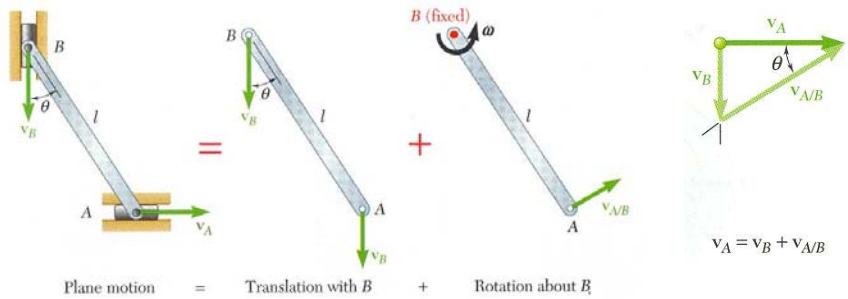
- Assuming that the velocity v_A of end *A* is known, wish to determine the velocity v_B of end *B* and the angular velocity ω in terms of v_A, *l*, and θ.
- The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

$$\frac{v_B}{v_A} = \tan\theta \qquad\qquad \frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} = \cos\theta$$

$$v_B = v_A \tan\theta \qquad\qquad \omega = \frac{v_A}{l\cos\theta}$$

Vector Mechanics for Engineers: Dynamics

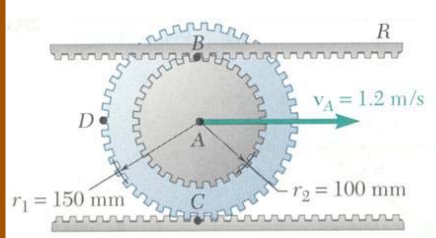
Absolute and Relative Velocity in Plane Motion



- Selecting point B as the reference point and solving for the velocity v_A of end A and the angular velocity ω leads to an equivalent velocity triangle.
- $v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point.
- Angular velocity ω of the rod in its rotation about B is the same as its rotation about A. Angular velocity is not dependent on the choice of reference point.

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.2



The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s.

Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack R and point D of the gear.

SOLUTION:

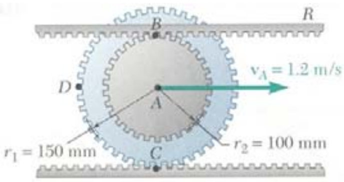
- The displacement of the gear center in one revolution is equal to the outer circumference. Relate the translational and angular displacements. Differentiate to relate the translational and angular velocities.
- The velocity for any point P on the gear may be written as

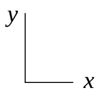
$$\vec{v}_P = \vec{v}_A + \vec{v}_{P/A} = \vec{v}_A + \omega\vec{k} \times \vec{r}_{P/A}$$

Evaluate the velocities of points B and D.

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.2





SOLUTION:

- The displacement of the gear center in one revolution is equal to the outer circumference.

  For $x_A > 0$ (moves to right), $\omega < 0$ (rotates clockwise).

$$\frac{x_A}{2\pi r} = -\frac{\theta}{2\pi} \qquad x_A = -r_1\theta$$

Differentiate to relate the translational and angular velocities.

$$v_A = -r_1\omega$$

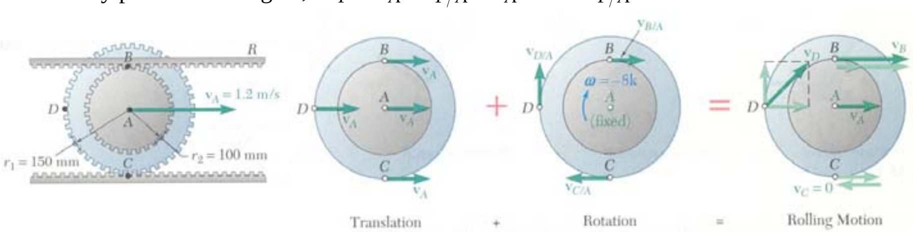
$$\omega = -\frac{v_A}{r_1} = -\frac{1.2\,\mathrm{m/s}}{0.150\,\mathrm{m}}$$

$$\boxed{\vec{\omega} = \omega\vec{k} = -(8\,\mathrm{rad/s})\vec{k}}$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.2

- For any point P on the gear, $\vec{v}_P = \vec{v}_A + \vec{v}_{P/A} = \vec{v}_A + \omega\vec{k} \times \vec{r}_{P/A}$



Velocity of the upper rack is equal to velocity of point B:

$$\vec{v}_R = \vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$

$$= (1.2\,\mathrm{m/s})\vec{i} + (8\,\mathrm{rad/s})\vec{k} \times (0.10\,\mathrm{m})\vec{j}$$

$$= (1.2\,\mathrm{m/s})\vec{i} + (0.8\,\mathrm{m/s})\vec{i}$$

$$\boxed{\vec{v}_R = (2\,\mathrm{m/s})\vec{i}}$$

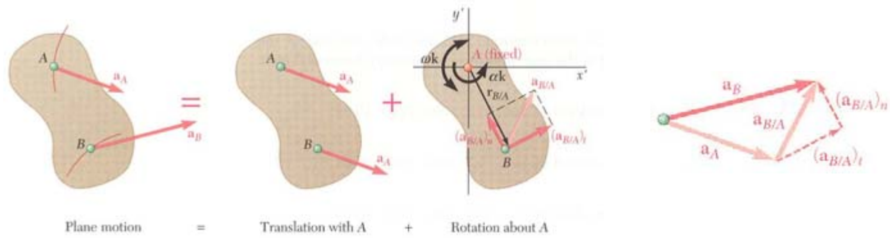
Velocity of the point D:

$$\vec{v}_D = \vec{v}_A + \omega\vec{k} \times \vec{r}_{D/A}$$

$$= (1.2\,\mathrm{m/s})\vec{i} + (8\,\mathrm{rad/s})\vec{k} \times (-0.150\,\mathrm{m})\vec{i}$$

$$\boxed{\begin{aligned}\vec{v}_D &= (1.2\,\mathrm{m/s})\vec{i} + (1.2\,\mathrm{m/s})\vec{j} \\ v_D &= 1.697\,\mathrm{m/s}\end{aligned}}$$

Absolute and Relative Acceleration in Plane Motion



- Absolute acceleration of a particle of the slab,

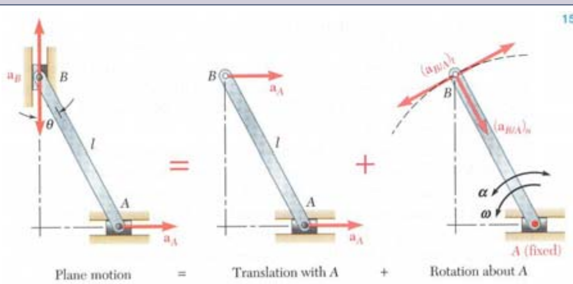
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

- Relative acceleration $\vec{a}_{B/A}$ associated with rotation about A includes tangential and normal components,

$$\left(\vec{a}_{B/A}\right)_t = \alpha\vec{k} \times \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_t = r\alpha$$

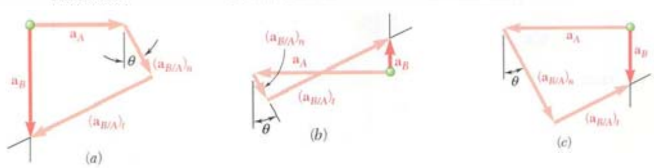
$$\left(\vec{a}_{B/A}\right)_n = -\omega^2\vec{r}_{B/A} \qquad \left(a_{B/A}\right)_n = r\omega^2$$

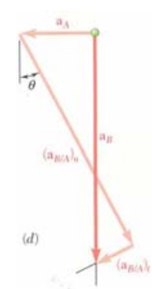
Absolute and Relative Acceleration in Plane Motion



- Given $\vec{a}_A$ and $\vec{v}_A$, determine $\vec{a}_B$ and $\vec{\alpha}$.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
$$= \vec{a}_A + \left(\vec{a}_{B/A}\right)_n + \left(\vec{a}_{B/A}\right)_t$$



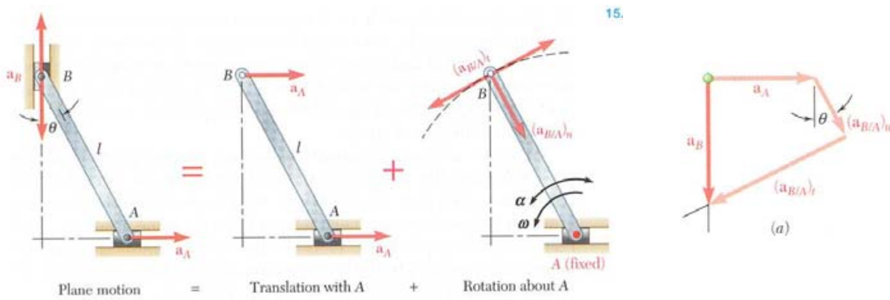


- Vector result depends on sense of $\vec{a}_A$ and the relative magnitudes of a_A and $\left(a_{B/A}\right)_n$

- Must also know angular velocity ω.

Vector Mechanics for Engineers: Dynamics

Absolute and Relative Acceleration in Plane Motion



- Write $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ in terms of the two component equations,

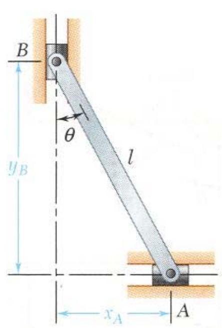
$\overset{+}{\rightarrow}$ x components: $0 = a_A + l\omega^2 \sin\theta - l\alpha\cos\theta$

$+\uparrow$ y components: $-a_B = -l\omega^2 \cos\theta - l\alpha\sin\theta$

- Solve for a_B and α.

Vector Mechanics for Engineers: Dynamics

Analysis of Plane Motion in Terms of a Parameter



- In some cases, it is advantageous to determine the absolute velocity and acceleration of a mechanism directly.

$$x_A = l\sin\theta \qquad y_B = l\cos\theta$$

$$v_A = \dot{x}_A = l\dot{\theta}\cos\theta = l\omega\cos\theta \qquad v_B = \dot{y}_B = -l\dot{\theta}\sin\theta = -l\omega\sin\theta$$

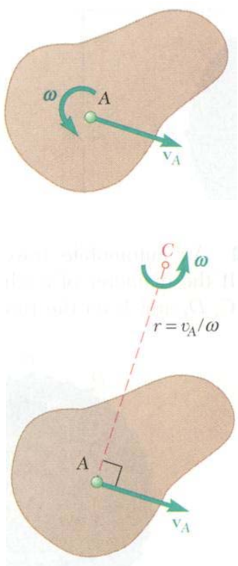
$$a_A = \ddot{x}_A = -l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta = -l\omega^2\sin\theta + l\alpha\cos\theta$$

$$a_B = \ddot{y}_B = -l\dot{\theta}^2\cos\theta - l\ddot{\theta}\sin\theta = -l\omega^2\cos\theta - l\alpha\sin\theta$$

Seventh Edition

Vector Mechanics for Engineers: Dynamics

Instantaneous Center of Rotation in Plane Motion

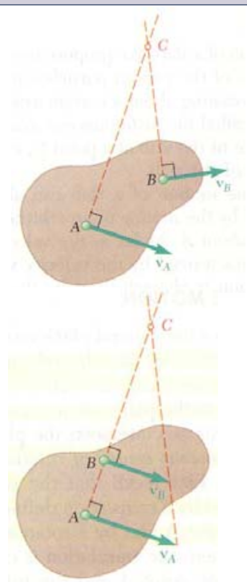


- Plane motion of all particles in a slab can always be replaced by the translation of an arbitrary point *A* and a rotation about *A* with an angular velocity that is independent of the choice of *A*.
- The same translational and rotational velocities at *A* are obtained by allowing the slab to rotate with the same angular velocity about the point *C* on a perpendicular to the velocity at *A*.
- The velocity of all other particles in the slab are the same as originally defined since the angular velocity and translational velocity at *A* are equivalent.
- As far as the velocities are concerned, the slab seems to rotate about the *instantaneous center of rotation C*.

Seventh Edition

Vector Mechanics for Engineers: Dynamics

Instantaneous Center of Rotation in Plane Motion

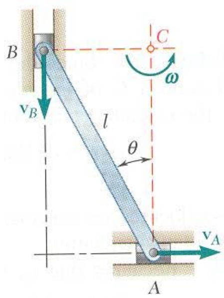


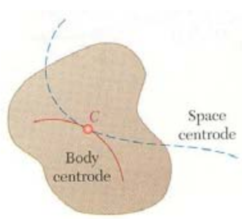
- If the velocity at two points *A* and *B* are known, the instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .
- If the velocity vectors are parallel, the instantaneous center of rotation is at infinity and the angular velocity is zero.
- If the velocity vectors at *A* and *B* are perpendicular to the line *AB*, the instantaneous center of rotation lies at the intersection of the line *AB* with the line joining the extremities of the velocity vectors at *A* and *B*.
- If the velocity magnitudes are equal, the instantaneous center of rotation is at infinity and the angular velocity is zero.

Vector Mechanics for Engineers: Dynamics

Seventh Edition

Instantaneous Center of Rotation in Plane Motion





- The instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .

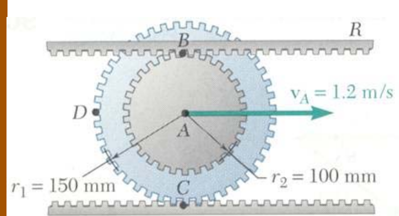
$$\omega = \frac{v_A}{AC} = \frac{v_A}{l\cos\theta} \qquad v_B = (BC)\omega = (l\sin\theta)\frac{v_A}{l\cos\theta} = v_A\tan\theta$$

- The velocities of all particles on the rod are as if they were rotated about *C.*
- The particle at the center of rotation has zero velocity.
- The particle coinciding with the center of rotation changes with time and the acceleration of the particle at the instantaneous center of rotation is not zero.
- The acceleration of the particles in the slab cannot be determined as if the slab were simply rotating about *C.*
- The trace of the locus of the center of rotation on the body is the body centrode and in space is the space centrode.

Vector Mechanics for Engineers: Dynamics

Seventh Edition

Sample Problem 15.4



The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s.

Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack *R* and point *D* of the gear.

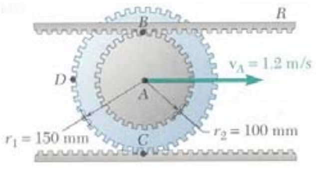
SOLUTION:

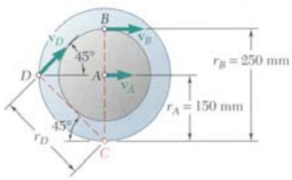
- The point *C* is in contact with the stationary lower rack and, instantaneously, has zero velocity. It must be the location of the instantaneous center of rotation.
- Determine the angular velocity about *C* based on the given velocity at *A.*
- Evaluate the velocities at *B* and *D* based on their rotation about *C.*

Vector Mechanics for Engineers: Dynamics

Seventh Edition

Sample Problem 15.4





SOLUTION:

- The point *C* is in contact with the stationary lower rack and, instantaneously, has zero velocity. It must be the location of the instantaneous center of rotation.
- Determine the angular velocity about *C* based on the given velocity at *A*.

$$v_A = r_A\omega \qquad \omega = \frac{v_A}{r_A} = \frac{1.2\,\text{m/s}}{0.15\,\text{m}} = 8\,\text{rad/s}$$

- Evaluate the velocities at *B* and *D* based on their rotation about *C*.

$$v_R = v_B = r_B\omega = (0.25\,\text{m})(8\,\text{rad/s})$$

$$\boxed{\vec{v}_R = (2\,\text{m/s})\vec{i}}$$

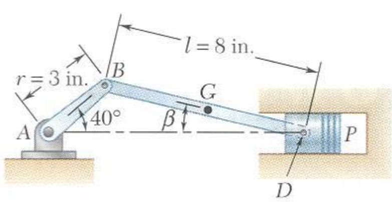
$$r_D = (0.15\,\text{m})\sqrt{2} = 0.2121\,\text{m}$$
$$v_D = r_D\omega = (0.2121\,\text{m})(8\,\text{rad/s})$$

$$\boxed{\begin{aligned} v_D &= 1.697\,\text{m/s} \\ \vec{v}_D &= (1.2\vec{i} + 1.2\vec{j})(\text{m/s}) \end{aligned}}$$

Vector Mechanics for Engineers: Dynamics

Seventh Edition

Sample Problem 15.5



The crank *AB* has a constant clockwise angular velocity of 2000 rpm.

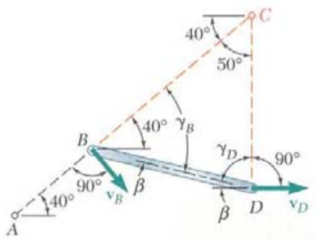
For the crank position indicated, determine *(a)* the angular velocity of the connecting rod *BD*, and *(b)* the velocity of the piston *P*.

SOLUTION:

- Determine the velocity at *B* from the given crank rotation data.
- The direction of the velocity vectors at *B* and *D* are known. The instantaneous center of rotation is at the intersection of the perpendiculars to the velocities through *B* and *D*.
- Determine the angular velocity about the center of rotation based on the velocity at *B*.
- Calculate the velocity at *D* based on its rotation about the instantaneous center of rotation.

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.5



$$\gamma_B = 40° + \beta = 53.95°$$
$$\gamma_D = 90° - \beta = 76.05°$$

$$\frac{BC}{\sin 76.05°} = \frac{CD}{\sin 53.95°} = \frac{8\,\text{in.}}{\sin 50°}$$

$$BC = 10.14\,\text{in.} \quad CD = 8.44\,\text{in.}$$

SOLUTION:

- From Sample Problem 15.3,

$$\vec{v}_B = \left(403.9\vec{i} - 481.3\vec{j}\right)(\text{in./s}) \qquad v_B = 628.3\,\text{in./s}$$
$$\beta = 13.95°$$

- The instantaneous center of rotation is at the intersection of the perpendiculars to the velocities through *B* and *D*.

- Determine the angular velocity about the center of rotation based on the velocity at *B*.

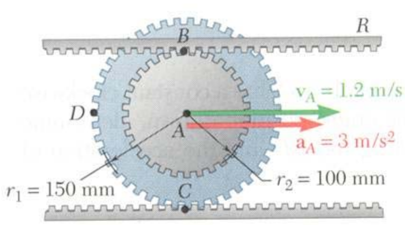
$$v_B = (BC)\omega_{BD}$$
$$\omega_{BD} = \frac{v_B}{BC} = \frac{628.3\,\text{in./s}}{10.14\,\text{in.}} \qquad \boxed{\omega_{BD} = 62.0\,\text{rad/s}}$$

- Calculate the velocity at *D* based on its rotation about the instantaneous center of rotation.

$$v_D = (CD)\omega_{BD} = (8.44\,\text{in.})(62.0\,\text{rad/s})$$
$$\boxed{v_P = v_D = 523\,\text{in./s} = 43.6\,\text{ft/s}}$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.6



The center of the double gear has a velocity and acceleration to the right of 1.2 m/s and 3 m/s^2, respectively. The lower rack is stationary.

Determine *(a)* the angular acceleration of the gear, and *(b)* the acceleration of points *B*, *C*, and *D*.

SOLUTION:

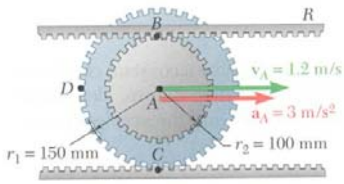
- The expression of the gear position as a function of θ is differentiated twice to define the relationship between the translational and angular accelerations.

- The acceleration of each point on the gear is obtained by adding the acceleration of the gear center and the relative accelerations with respect to the center. The latter includes normal and tangential acceleration components.

Seventh Edition

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.6



SOLUTION:

- The expression of the gear position as a function of θ is differentiated twice to define the relationship between the translational and angular accelerations.

$$x_A = -r_1\theta$$

$$v_A = -r_1\dot{\theta} = -r_1\omega$$

$$\omega = -\frac{v_A}{r_1} = -\frac{1.2\,\mathrm{m/s}}{0.150\,\mathrm{m}} = -8\,\mathrm{rad/s}$$

$$a_A = -r_1\ddot{\theta} = -r_1\alpha$$

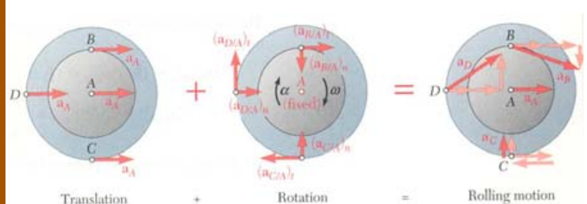
$$\alpha = -\frac{a_A}{r_1} = -\frac{3\,\mathrm{m/s^2}}{0.150\,\mathrm{m}}$$

$$\boxed{\vec{\alpha} = \alpha\vec{k} = -\left(20\,\mathrm{rad/s^2}\right)\vec{k}}$$

Seventh Edition

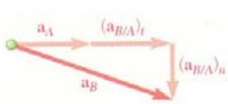
Vector Mechanics for Engineers: Dynamics

Sample Problem 15.6



- The acceleration of each point is obtained by adding the acceleration of the gear center and the relative accelerations with respect to the center.

  The latter includes normal and tangential acceleration components.



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \left(\vec{a}_{B/A}\right)_t + \left(\vec{a}_{B/A}\right)_n$$

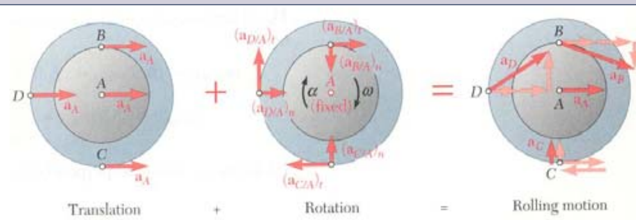
$$= \vec{a}_A + \alpha\vec{k}\times\vec{r}_{B/A} - \omega^2\vec{r}_{B/A}$$

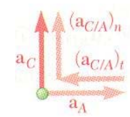
$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(20\,\mathrm{rad/s^2}\right)\vec{k}\times(0.100\,\mathrm{m})\vec{j} - (8\,\mathrm{rad/s})^2(-0.100\,\mathrm{m})\vec{j}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} + \left(2\,\mathrm{m/s^2}\right)\vec{i} - \left(6.40\,\mathrm{m/s^2}\right)\vec{j}$$

$$\boxed{\vec{a}_B = \left(5\,\mathrm{m/s^2}\right)\vec{i} - \left(6.40\,\mathrm{m/s^2}\right)\vec{j} \qquad a_B = 8.12\,\mathrm{m/s^2}}$$

Sample Problem 15.6



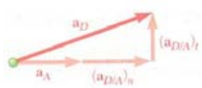


$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A} = \vec{a}_A + \alpha\vec{k}\times\vec{r}_{C/A} - \omega^2\vec{r}_{C/A}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(20\,\mathrm{rad/s^2}\right)\vec{k}\times\left(-0.150\,\mathrm{m}\right)\vec{j} - \left(8\,\mathrm{rad/s}\right)^2\left(-0.150\,\mathrm{m}\right)\vec{j}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(3\,\mathrm{m/s^2}\right)\vec{i} + \left(9.60\,\mathrm{m/s^2}\right)\vec{j}$$

$$\boxed{\vec{a}_c = \left(9.60\,\mathrm{m/s^2}\right)\vec{j}}$$



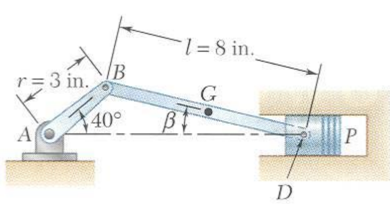
$$\vec{a}_D = \vec{a}_A + \vec{a}_{D/A} = \vec{a}_A + \alpha\vec{k}\times\vec{r}_{D/A} - \omega^2\vec{r}_{D/A}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(20\,\mathrm{rad/s^2}\right)\vec{k}\times\left(-0.150\,\mathrm{m}\right)\vec{i} - \left(8\,\mathrm{rad/s}\right)^2\left(-0.150\,\mathrm{m}\right)\vec{i}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} + \left(3\,\mathrm{m/s^2}\right)\vec{j} + \left(9.60\,\mathrm{m/s^2}\right)\vec{i}$$

$$\boxed{\vec{a}_D = \left(12.6\,\mathrm{m/s^2}\right)\vec{i} + \left(3\,\mathrm{m/s^2}\right)\vec{j} \qquad a_D = 12.95\,\mathrm{m/s^2}}$$

Sample Problem 15.3



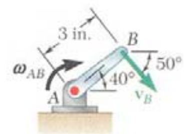
The crank *AB* has a constant clockwise angular velocity of 2000 rpm.

For the crank position indicated, determine *(a)* the angular velocity of the connecting rod *BD*, and *(b)* the velocity of the piston *P*.

SOLUTION:

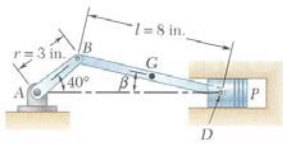
- Will determine the absolute velocity of point *D* with
$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$
- The velocity $\vec{v}_B$ is obtained from the given crank rotation data.
- The directions of the absolute velocity $\vec{v}_D$ and the relative velocity $\vec{v}_{D/B}$ are determined from the problem geometry.
- The unknowns in the vector expression are the velocity magnitudes v_D and $v_{D/B}$ which may be determined from the corresponding vector triangle.
- The angular velocity of the connecting rod is calculated from $v_{D/B}$.

Sample Problem 15.3



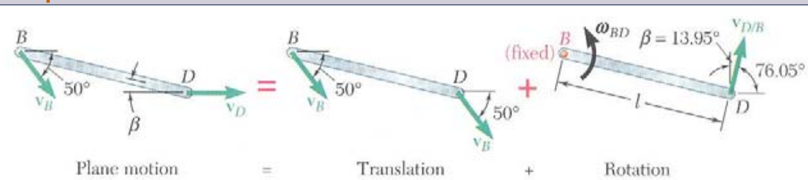
SOLUTION:

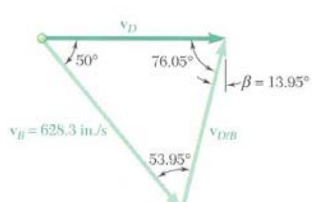
- Will determine the absolute velocity of point D with
  $$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$
- The velocity $\vec{v}_B$ is obtained from the crank rotation data.
  $$\omega_{AB} = \left(2000\frac{\text{rev}}{\text{min}}\right)\left(\frac{\text{min}}{60\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right) = 209.4\ \text{rad/s}$$
  $$v_B = (AB)\omega_{AB} = (3\text{in.})(209.4\ \text{rad/s})$$
  The velocity direction is as shown.



- The direction of the absolute velocity $\vec{v}_D$ is horizontal. The direction of the relative velocity $\vec{v}_{D/B}$ is perpendicular to BD. Compute the angle between the horizontal and the connecting rod from the law of sines.
  $$\frac{\sin 40°}{8\text{in.}} = \frac{\sin\beta}{3\text{in.}} \qquad \beta = 13.95°$$

Sample Problem 15.3



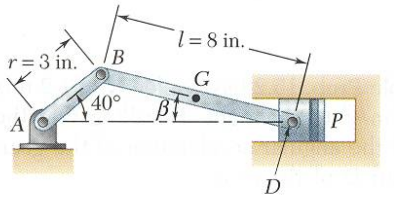


$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

- Determine the velocity magnitudes v_D and $v_{D/B}$ from the vector triangle.
  $$\frac{v_D}{\sin 53.95°} = \frac{v_{D/B}}{\sin 50°} = \frac{628.3\text{in./s}}{\sin 76.05°}$$
  $$v_D = 523.4\text{in./s} = 43.6\,\text{ft/s} \qquad \boxed{v_P = v_D = 43.6\,\text{ft/s}}$$
  $$v_{D/B} = 495.9\text{in./s}$$
  $$v_{D/B} = l\omega_{BD}$$
  $$\omega_{BD} = \frac{v_{D/B}}{l} = \frac{495.9\text{in./s}}{8\,\text{in.}}$$
  $$= 62.0\ \text{rad/s} \qquad \boxed{\vec{\omega}_{BD} = (62.0\ \text{rad/s})\vec{k}}$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.7



Crank *AG* of the engine system has a constant clockwise angular velocity of 2000 rpm.

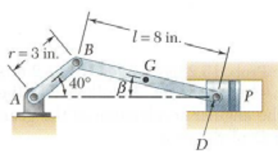
For the crank position shown, determine the angular acceleration of the connecting rod *BD* and the acceleration of point *D*.

SOLUTION:

- The angular acceleration of the connecting rod *BD* and the acceleration of point *D* will be determined from
  $\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = \vec{a}_B + \left(\vec{a}_{D/B}\right)_t + \left(\vec{a}_{D/B}\right)_n$
- The acceleration of *B* is determined from the given rotation speed of *AB*.
- The directions of the accelerations $\vec{a}_D$, $\left(\vec{a}_{D/B}\right)_t$, and $\left(\vec{a}_{D/B}\right)_n$ are determined from the geometry.
- Component equations for acceleration of point *D* are solved simultaneously for acceleration of *D* and angular acceleration of the connecting rod.

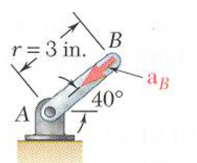
Vector Mechanics for Engineers: Dynamics

Sample Problem 15.7



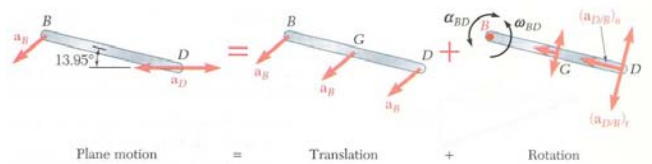
SOLUTION:

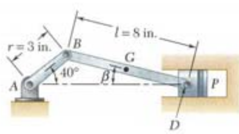
- The angular acceleration of the connecting rod *BD* and the acceleration of point *D* will be determined from
  $$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = \vec{a}_B + \left(\vec{a}_{D/B}\right)_t + \left(\vec{a}_{D/B}\right)_n$$
- The acceleration of *B* is determined from the given rotation speed of *AB*.



$$\omega_{AB} = 2000\,\text{rpm} = 209.4\,\text{rad/s} = \text{constant}$$
$$\alpha_{AB} = 0$$
$$a_B = r\omega_{AB}^2 = \left(\tfrac{3}{12}\,\text{ft}\right)(209.4\,\text{rad/s})^2 = 10{,}962\,\text{ft/s}^2$$
$$\vec{a}_B = \left(10{,}962\,\text{ft/s}^2\right)\left(-\cos 40°\,\vec{i} - \sin 40°\,\vec{j}\right)$$

Sample Problem 15.7





- The directions of the accelerations $\vec{a}_D, \left(\vec{a}_{D/B}\right)_t$, and $\left(\vec{a}_{D/B}\right)_n$ are determined from the geometry.

$$\vec{a}_D = \mp a_D \vec{i}$$

From Sample Problem 15.3, $\omega_{BD} = 62.0$ rad/s, $\beta = 13.95°$.

$$\left(a_{D/B}\right)_n = (BD)\omega_{BD}^2 = \left(\tfrac{8}{12}\,\text{ft}\right)(62.0\,\text{rad/s})^2 = 2563\,\text{ft/s}^2$$

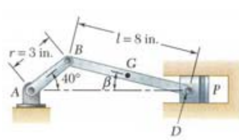
$$\left(\vec{a}_{D/B}\right)_n = \left(2563\,\text{ft/s}^2\right)\left(-\cos 13.95°\vec{i} + \sin 13.95°\vec{j}\right)$$

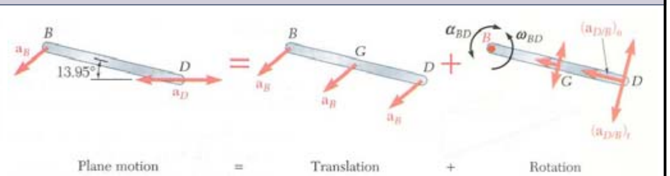
$$\left(a_{D/B}\right)_t = (BD)\alpha_{BD} = \left(\tfrac{8}{12}\,\text{ft}\right)\alpha_{BD} = 0.667\alpha_{BD}$$

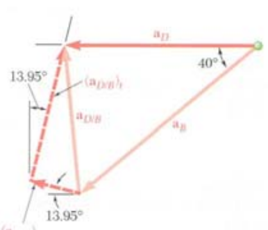
The direction of $(a_{D/B})_t$ is known but the sense is not known,

$$\left(\vec{a}_{D/B}\right)_t = \left(0.667\alpha_{BD}\right)\left(\pm \sin 76.05°\vec{i} \pm \cos 76.05°\vec{j}\right)$$

Sample Problem 15.7







- Component equations for acceleration of point D are solved simultaneously.

$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = \vec{a}_B + \left(\vec{a}_{D/B}\right)_t + \left(\vec{a}_{D/B}\right)_n$$

x components:

$$-a_D = -10{,}962\cos 40° - 2563\cos 13.95° + 0.667\alpha_{BD}\sin 13.95°$$

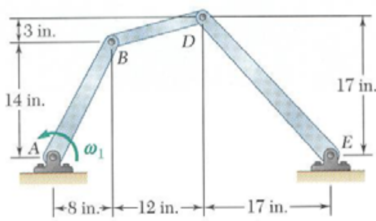
y components:

$$0 = -10{,}962\sin 40° + 2563\sin 13.95° + 0.667\alpha_{BD}\cos 13.95°$$

$$\boxed{\begin{aligned}\vec{\alpha}_{BD} &= \left(9940\,\text{rad/s}^2\right)\vec{k}\\ \vec{a}_D &= -\left(9290\,\text{ft/s}^2\right)\vec{i}\end{aligned}}$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.8



In the position shown, crank *AB* has a constant angular velocity ω_1 = 20 rad/s counterclockwise.

Determine the angular velocities and angular accelerations of the connecting rod *BD* and crank *DE*.

SOLUTION:

- The angular velocities are determined by simultaneously solving the component equations for

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

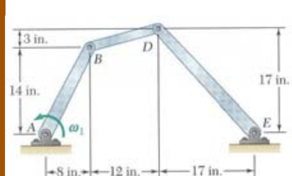
- The angular accelerations are determined by simultaneously solving the component equations for

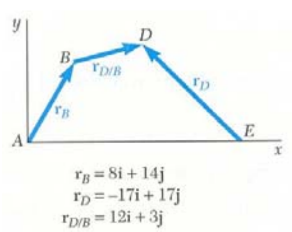
$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$$

Seventh Edition

Vector Mechanics for Engineers: Dynamics

Sample Problem 15.8





SOLUTION:

- The angular velocities are determined by simultaneously solving the component equations for

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

$$\begin{aligned}\vec{v}_D &= \vec{\omega}_{DE} \times \vec{r}_D = \omega_{DE}\vec{k} \times \left(-17\vec{i} + 17\vec{j}\right)\\ &= -17\omega_{DE}\vec{i} - 17\omega_{DE}\vec{j}\end{aligned}$$

$$\begin{aligned}\vec{v}_B &= \vec{\omega}_{AB} \times \vec{r}_B = 20\vec{k} \times \left(8\vec{i} + 14\vec{j}\right)\\ &= -280\vec{i} + 160\vec{j}\end{aligned}$$

$$\begin{aligned}\vec{v}_{D/B} &= \vec{\omega}_{BD} \times \vec{r}_{D/B} = \omega_{BD}\vec{k} \times \left(12\vec{i} + 3\vec{j}\right)\\ &= -3\omega_{BD}\vec{i} + 12\omega_{BD}\vec{j}\end{aligned}$$

x components: $-17\omega_{DE} = -280 - 3\omega_{BD}$

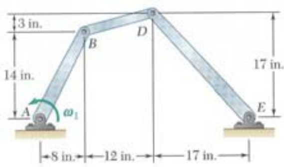
y components: $-17\omega_{DE} = +160 + 12\omega_{BD}$

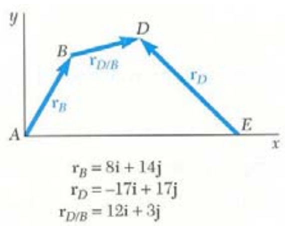
$$\boxed{\vec{\omega}_{BD} = -(29.33\,\mathrm{rad/s})\vec{k} \qquad \vec{\omega}_{DE} = (11.29\,\mathrm{rad/s})\vec{k}}$$

Vector Mechanics for Engineers: Dynamics

Seventh Edition

Sample Problem 15.8





- The angular accelerations are determined by simultaneously solving the component equations for

$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$$

$$\vec{a}_D = \vec{\alpha}_{DE} \times \vec{r}_D - \omega_{DE}^2 \vec{r}_D$$
$$= \alpha_{DE}\vec{k} \times \left(-17\vec{i} + 17\vec{j}\right) - (11.29)^2\left(-17\vec{i} + 17\vec{j}\right)$$
$$= -17\alpha_{DE}\vec{i} - 17\alpha_{DE}\vec{j} + 2170\vec{i} - 2170\vec{j}$$

$$\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_B - \omega_{AB}^2 \vec{r}_B = 0 - (20)^2\left(8\vec{i} + 14\vec{j}\right)$$
$$= -3200\vec{i} + 5600\vec{j}$$

$$\vec{a}_{D/B} = \vec{\alpha}_{BD} \times \vec{r}_{B/D} - \omega_{BD}^2 \vec{r}_{B/D}$$
$$= \alpha_{B/D}\vec{k} \times \left(12\vec{i} + 3\vec{j}\right) - (29.33)^2\left(12\vec{i} + 3\vec{j}\right)$$
$$= -3\alpha_{B/D}\vec{i} + 12\alpha_{B/D}\vec{j} - 10{,}320\vec{i} - 2580\vec{j}$$

x components: $\quad -17\alpha_{DE} + 3\alpha_{BD} = -15{,}690$

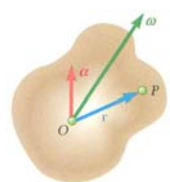
y components: $\quad -17\alpha_{DE} - 12\alpha_{BD} = -6010$

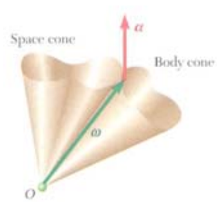
$$\boxed{\vec{\alpha}_{BD} = -\left(645\,\mathrm{rad/s^2}\right)\vec{k} \qquad \vec{\alpha}_{DE} = \left(809\,\mathrm{rad/s^2}\right)\vec{k}}$$

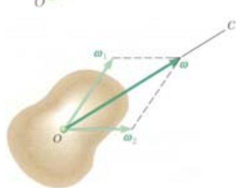
Vector Mechanics for Engineers: Dynamics

Seventh Edition

Motion About a Fixed Point







- The most general displacement of a rigid body with a fixed point *O* is equivalent to a rotation of the body about an axis through *O*.
- With the instantaneous axis of rotation and angular velocity $\vec{\omega}$, the velocity of a particle *P* of the body is

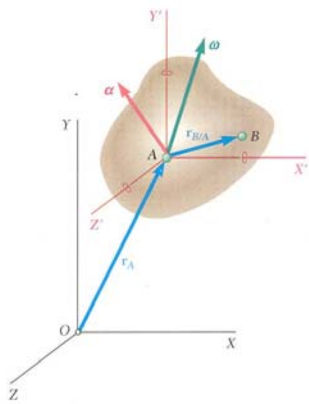
$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$

and the acceleration of the particle *P* is

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right) \qquad \vec{\alpha} = \frac{d\vec{\omega}}{dt}.$$

- The angular acceleration $\vec{\alpha}$ represents the velocity of the tip of $\vec{\omega}$.
- As the vector $\vec{\omega}$ moves within the body and in space, it generates a body cone and space cone which are tangent along the instantaneous axis of rotation.
- Angular velocities have magnitude and direction and obey parallelogram law of addition. They are vectors.

General Motion



- For particles A and B of a rigid body,

  $$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

- Particle A is fixed within the body and motion of the body relative to $AX'Y'Z'$ is the motion of a body with a fixed point

  $$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

- Similarly, the acceleration of the particle P is

  $$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
  $$= \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

- Most general motion of a rigid body is equivalent to:
  - a translation in which all particles have the same velocity and acceleration of a reference particle A, and
  - of a motion in which particle A is assumed fixed.