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CHAPTER 13

VECTOR MECHANICS FOR ENGINEERS:

DYNAMICS

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Lecture Notes:
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Kinetics of Particles: Energy and Momentum Methods

Introduction

- Previously, problems dealing with the motion of particles were solved through the fundamental equation of motion, $\vec{F} = m\vec{a}$. Current chapter introduces two additional methods of analysis.

- *Method of work and energy*: directly relates force, mass, velocity and displacement.

- *Method of impulse and momentum*: directly relates force, mass, velocity, and time.

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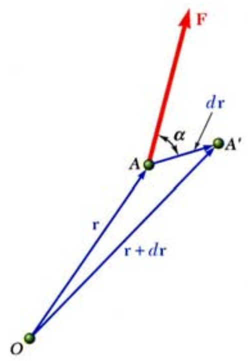
Work of a Force



- Differential vector $d\vec{r}$ is the *particle displacement*.
- *Work of the force* is

$$
\begin{aligned}
dU &= \vec{F} \bullet d\vec{r} \\
&= F\,ds\,\cos\alpha \\
&= F_x dx + F_y dy + F_z dz
\end{aligned}
$$

- Work is a scalar quantity, i.e., it has magnitude and sign but not direction.
- Dimensions of work are length $\times$ force. Units are

$$1\text{ J }(\textit{joule}) = (1\text{ N})(1\text{ m}) \qquad 1\text{ ft}\cdot\text{lb} = 1.356\text{ J}$$

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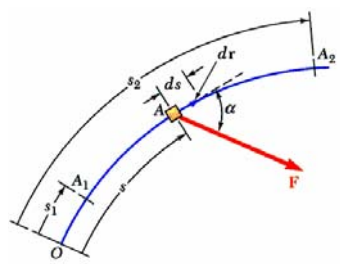
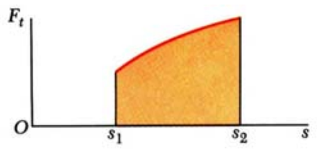
Work of a Force



- Work of a force during a finite displacement,

$$
\begin{aligned}
U_{1\to 2} &= \int_{A_1}^{A_2} \vec{F} \bullet d\vec{r} \\
&= \int_{s_1}^{s_2} (F\cos\alpha)\,ds = \int_{s_1}^{s_2} F_t\,ds \\
&= \int_{A_1}^{A_2} \left(F_x dx + F_y dy + F_z dz\right)
\end{aligned}
$$



- Work is represented by the area under the curve of F_t plotted against s.

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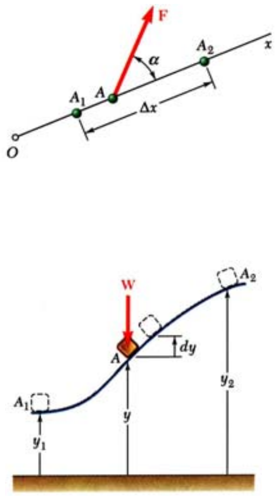
Work of a Force



- Work of a constant force in rectilinear motion,

$$U_{1\to2} = (F\cos\alpha)\,\Delta x$$

- Work of the force of gravity,

$$dU = F_x dx + F_y dy + F_z dz$$
$$= -W\,dy$$
$$U_{1\to2} = -\int_{y_1}^{y_2} W\,dy$$
$$= -W(y_2 - y_1) = -W\,\Delta y$$

- Work *of the weight* is equal to product of weight W and vertical displacement Δy.
- Work *of the weight* is positive when $\Delta y < 0$, i.e., when the weight moves down.

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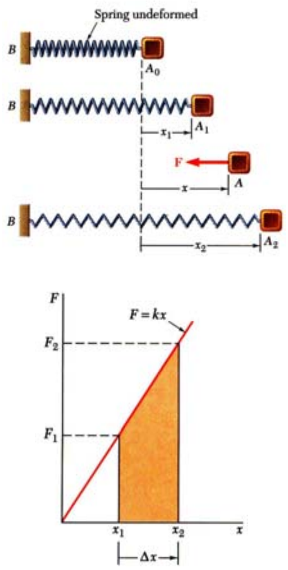
Work of a Force



- Magnitude of the force exerted by a spring is proportional to deflection,

$$F = kx$$
$$k = \text{spring constant (N/m or lb/in.)}$$

- Work *of the force exerted by spring*,

$$dU = -F\,dx = -kx\,dx$$
$$U_{1\to2} = -\int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

- Work *of the force exerted by spring* is positive when $x_2 < x_1$, i.e., when the spring is returning to its undeformed position.
- Work of the force exerted by the spring is equal to negative of area under curve of F plotted against x,

$$U_{1\to2} = -\tfrac{1}{2}(F_1 + F_2)\,\Delta x$$

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Work of a Force

Forces which *do not* do work ($ds = 0$ or $\cos\alpha = 0$):

- reaction at frictionless pin supporting rotating body,
- reaction at frictionless surface when body in contact moves along surface,
- reaction at a roller moving along its track, and
- weight of a body when its center of gravity moves horizontally.

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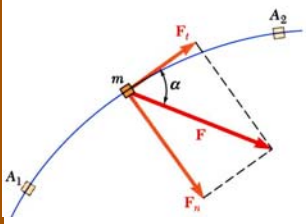
Particle Kinetic Energy: Principle of Work & Energy



- Consider a particle of mass m acted upon by force $\vec{F}$

$$F_t = ma_t = m\frac{dv}{dt}$$

$$= m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds}$$

$$F_t\,ds = mv\,dv$$

- Integrating from A_1 to A_2 ,

$$\int_{s_1}^{s_2} F_t ds = m\int_{v_1}^{v_2} v\,dv = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$U_{1\to 2} = T_2 - T_1 \qquad T = \tfrac{1}{2}mv^2 = \textit{kinetic energy}$$

- *The work of the force* $\vec{F}$ *is equal to the change in kinetic energy of the particle.*
- Units of work and kinetic energy are the same:

$$T = \tfrac{1}{2}mv^2 = \mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)\mathrm{m} = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

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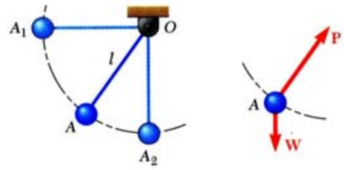
Applications of the Principle of Work and Energy



- Wish to determine velocity of pendulum bob at A_2. Consider work & kinetic energy.
- Force $\vec{P}$ acts normal to path and does no work.

$$T_1 + U_{1\to 2} = T_2$$

$$0 + Wl = \frac{1}{2}\frac{W}{g}v_2^2$$

$$v_2 = \sqrt{2gl}$$

- Velocity found without determining expression for acceleration and integrating.
- All quantities are scalars and can be added directly.
- Forces which do no work are eliminated from the problem.

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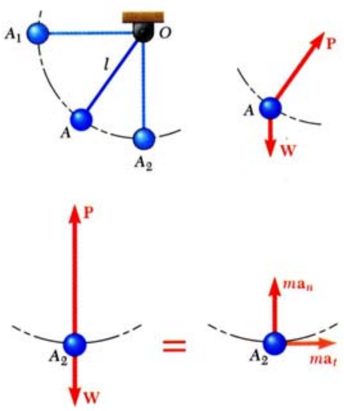
Applications of the Principle of Work and Energy



$$v_2 = \sqrt{2gl}$$

- Principle of work and energy cannot be applied to directly determine the acceleration of the pendulum bob.
- Calculating the tension in the cord requires supplementing the method of work and energy with an application of Newton's second law.
- As the bob passes through A_2 ,

$$\sum F_n = m a_n$$

$$P - W = \frac{W}{g}\frac{v_2^2}{l}$$

$$P = W + \frac{W}{g}\frac{2gl}{l} = 3W$$

Power and Efficiency

- *Power* = rate at which work is done.

$$= \frac{dU}{dt} = \frac{\vec{F} \bullet d\vec{r}}{dt}$$

$$= \vec{F} \bullet \vec{v}$$

- Dimensions of power are work/time or force*velocity. Units for power are

$$1\ \mathrm{W\ (watt)} = 1\frac{\mathrm{J}}{\mathrm{s}} = 1\ \mathrm{N} \cdot \frac{\mathrm{m}}{\mathrm{s}} \quad \text{or} \quad 1\ \mathrm{hp} = 550\frac{\mathrm{ft \cdot lb}}{\mathrm{s}} = 746\ \mathrm{W}$$

- η = efficiency

$$= \frac{\text{output work}}{\text{input work}}$$

$$= \frac{\text{power output}}{\text{power input}}$$

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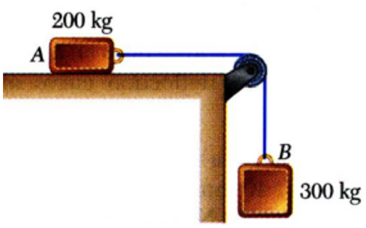
Sample Problem 13.2

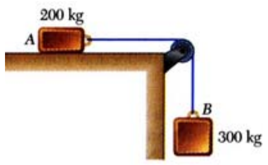


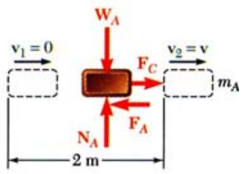
Two blocks are joined by an inextensible cable as shown. If the system is released from rest, determine the velocity of block *A* after it has moved 2 m. Assume that the coefficient of friction between block *A* and the plane is $\mu_k = 0.25$ and that the pulley is weightless and frictionless.

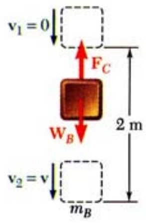
SOLUTION:

- Apply the principle of work and energy separately to blocks *A* and *B*.
- When the two relations are combined, the work of the cable forces cancel. Solve for the velocity.

Sample Problem 13.2







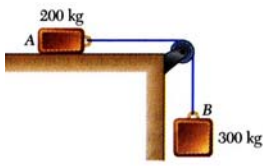
SOLUTION:

- Apply the principle of work and energy separately to blocks *A* and *B*.

$$W_A = (200\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 1962\,\mathrm{N}$$
$$F_A = \mu_k N_A = \mu_k W_A = 0.25(1962\,\mathrm{N}) = 490\,\mathrm{N}$$
$$T_1 + U_{1\to 2} = T_2 :$$
$$0 + F_C(2\,\mathrm{m}) - F_A(2\,\mathrm{m}) = \tfrac{1}{2} m_A v^2$$
$$F_C(2\,\mathrm{m}) - (490\,\mathrm{N})(2\,\mathrm{m}) = \tfrac{1}{2}(200\,\mathrm{kg})v^2$$

$$W_B = (300\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 2940\,\mathrm{N}$$
$$T_1 + U_{1\to 2} = T_2 :$$
$$0 - F_c(2\,\mathrm{m}) + W_B(2\,\mathrm{m}) = \tfrac{1}{2} m_B v^2$$
$$-F_c(2\,\mathrm{m}) + (2940\,\mathrm{N})(2\,\mathrm{m}) = \tfrac{1}{2}(300\,\mathrm{kg})v^2$$

Sample Problem 13.2



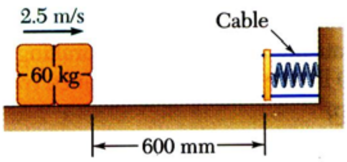
- When the two relations are combined, the work of the cable forces cancel. Solve for the velocity.

$$F_C(2\,\mathrm{m}) - (490\,\mathrm{N})(2\,\mathrm{m}) = \tfrac{1}{2}(200\,\mathrm{kg})v^2$$
$$-F_c(2\,\mathrm{m}) + (2940\,\mathrm{N})(2\,\mathrm{m}) = \tfrac{1}{2}(300\,\mathrm{kg})v^2$$

$$(2940\,\mathrm{N})(2\,\mathrm{m}) - (490\,\mathrm{N})(2\,\mathrm{m}) = \tfrac{1}{2}(200\,\mathrm{kg} + 300\,\mathrm{kg})v^2$$
$$4900\,\mathrm{J} = \tfrac{1}{2}(500\,\mathrm{kg})v^2$$

$$\boxed{v = 4.43\,\mathrm{m/s}}$$

Sample Problem 13.3



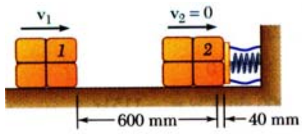
A spring is used to stop a 60 kg package which is sliding on a horizontal surface. The spring has a constant k = 20 kN/m and is held by cables so that it is initially compressed 120 mm. The package has a velocity of 2.5 m/s in the position shown and the maximum deflection of the spring is 40 mm.

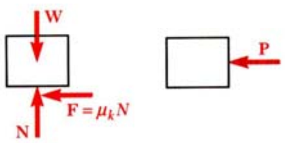
Determine *(a)* the coefficient of kinetic friction between the package and surface and *(b)* the velocity of the package as it passes again through the position shown.

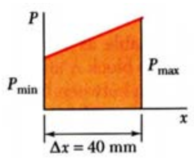
SOLUTION:

- Apply the principle of work and energy between the initial position and the point at which the spring is fully compressed and the velocity is zero. The only unknown in the relation is the friction coefficient.
- Apply the principle of work and energy for the rebound of the package. The only unknown in the relation is the velocity at the final position.

Sample Problem 13.3







SOLUTION:

- Apply principle of work and energy between initial position and the point at which spring is fully compressed.

$$T_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2}(60\,\text{kg})(2.5\,\text{m/s})^2 = 187.5\,\text{J} \qquad T_2 = 0$$

$$\begin{aligned}(U_{1\to2})_f &= -\mu_k W x \\ &= -\mu_k (60\,\text{kg})(9.81\,\text{m/s}^2)(0.640\,\text{m}) = -(377\,\text{J})\mu_k\end{aligned}$$

$$P_{min} = k x_0 = (20\,\text{kN/m})(0.120\,\text{m}) = 2400\,\text{N}$$

$$P_{max} = k(x_0 + \Delta x) = (20\,\text{kN/m})(0.160\,\text{m}) = 3200\,\text{N}$$

$$\begin{aligned}(U_{1\to2})_e &= -\tfrac{1}{2}(P_{min} + P_{max})\Delta x \\ &= -\tfrac{1}{2}(2400\,\text{N} + 3200\,\text{N})(0.040\,\text{m}) = -112.0\,\text{J}\end{aligned}$$

$$U_{1\to2} = (U_{1\to2})_f + (U_{1\to2})_e = -(377\,\text{J})\mu_k - 112\,\text{J}$$

$$T_1 + U_{1\to2} = T_2 :$$

$$187.5\,\text{J} - (377\,\text{J})\mu_k - 112\,\text{J} = 0 \qquad \boxed{\mu_k = 0.20}$$

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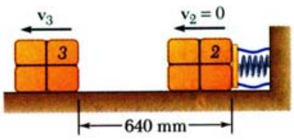
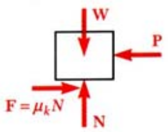
Sample Problem 13.3





- Apply the principle of work and energy for the rebound of the package.

$$T_2 = 0 \qquad T_3 = \tfrac{1}{2} m v_3^2 = \tfrac{1}{2}(60\,\mathrm{kg}) v_3^2$$

$$U_{2\to3} = (U_{2\to3})_f + (U_{2\to3})_e = -(377\,\mathrm{J})\mu_k + 112\,\mathrm{J}$$
$$= +36.5\,\mathrm{J}$$

$$T_2 + U_{2\to3} = T_3 :$$
$$0 + 36.5\,\mathrm{J} = \tfrac{1}{2}(60\,\mathrm{kg}) v_3^2$$

$$\boxed{v_3 = 1.103\,\mathrm{m/s}}$$

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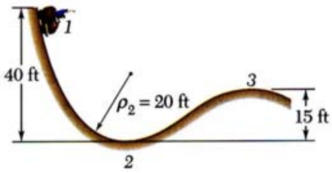
Sample Problem 13.4



A 2000 lb car starts from rest at point 1 and moves without friction down the track shown.

Determine:

a) the force exerted by the track on the car at point 2, and

b) the minimum safe value of the radius of curvature at point 3.

SOLUTION:

- Apply principle of work and energy to determine velocity at point 2.
- Apply Newton's second law to find normal force by the track at point 2.
- Apply principle of work and energy to determine velocity at point 3.
- Apply Newton's second law to find minimum radius of curvature at point 3 such that a positive normal force is exerted by the track.

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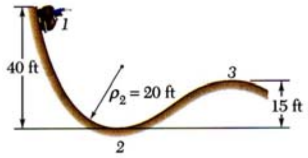
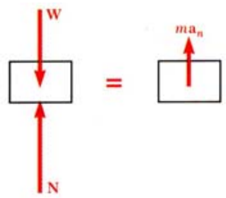
Sample Problem 13.4





SOLUTION:

- Apply principle of work and energy to determine velocity at point 2.

$$T_1 = 0 \qquad T_2 = \tfrac{1}{2}mv_2^2 = \frac{1}{2}\frac{W}{g}v_2^2$$

$$U_{1\to 2} = +W(40\,\text{ft})$$

$$T_1 + U_{1\to 2} = T_2: \qquad 0 + W(40\,\text{ft}) = \frac{1}{2}\frac{W}{g}v_2^2$$

$$v_2^2 = 2(40\,\text{ft})g = 2(40\,\text{ft})\left(32.2\,\text{ft}/s^2\right) \qquad v_2 = 50.8\,\text{ft/s}$$

- Apply Newton’s second law to find normal force by the track at point 2.

$$+\uparrow \sum F_n = ma_n:$$

$$-W + N = ma_n = \frac{W}{g}\frac{v_2^2}{\rho_2} = \frac{W}{g}\frac{2(40\,\text{ft})g}{20\,\text{ft}}$$

$$N = 5W \qquad \boxed{N = 10000\,\text{lb}}$$

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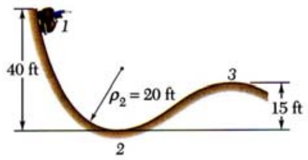
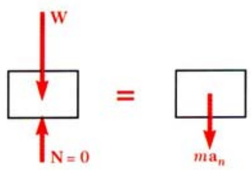
Sample Problem 13.4





- Apply principle of work and energy to determine velocity at point 3.

$$T_1 + U_{1\to 3} = T_3 \qquad 0 + W(25\,\text{ft}) = \frac{1}{2}\frac{W}{g}v_3^2$$

$$v_3^2 = 2(25\,\text{ft})g = 2(25\,\text{ft})(32.2\,\text{ft/s}) \qquad v_3 = 40.1\,\text{ft/s}$$

- Apply Newton’s second law to find minimum radius of curvature at point 3 such that a positive normal force is exerted by the track.

$$+\downarrow \sum F_n = ma_n:$$

$$W = ma_n$$

$$= \frac{W}{g}\frac{v_3^2}{\rho_3} = \frac{W}{g}\frac{2(25\,\text{ft})g}{\rho_3} \qquad \boxed{\rho_3 = 50\,\text{ft}}$$

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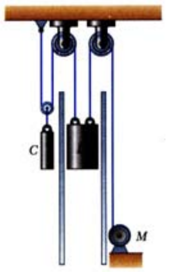
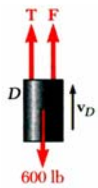
Sample Problem 13.5



The dumbwaiter *D* and its load have a combined weight of 600 lb, while the counterweight *C* weighs 800 lb.

Determine the power delivered by the electric motor *M* when the dumbwaiter *(a)* is moving up at a constant speed of 8 ft/s and *(b)* has an instantaneous velocity of 8 ft/s and an acceleration of 2.5 ft/s^2, both directed upwards.

SOLUTION:



Force exerted by the motor cable has same direction as the dumbwaiter velocity. Power delivered by motor is equal to Fv_D, v_D = 8 ft/s.

- In the first case, bodies are in uniform motion. Determine force exerted by motor cable from conditions for static equilibrium.
- In the second case, both bodies are accelerating. Apply Newton's second law to each body to determine the required motor cable force.

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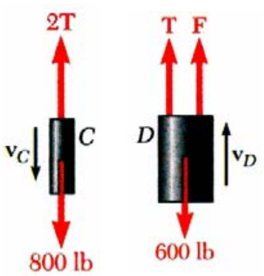
Sample Problem 13.5



- In the first case, bodies are in uniform motion. Determine force exerted by motor cable from conditions for static equilibrium.

Free-body C:

$$+\uparrow \sum F_y = 0: \quad 2T - 800\,\text{lb} = 0 \qquad T = 400\,\text{lb}$$

Free-body D:

$$+\uparrow \sum F_y = 0: \quad F + T - 600\,\text{lb} = 0$$

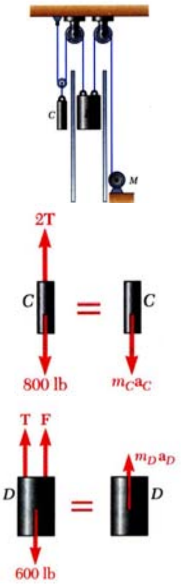
$$F = 600\,\text{lb} - T = 600\,\text{lb} - 400\,\text{lb} = 200\,\text{lb}$$

$$Power = Fv_D = (200\,\text{lb})(8\,\text{ft/s})$$

$$= 1600\,\text{ft}\cdot\text{lb/s}$$

$$\boxed{Power = (1600\,\text{ft}\cdot\text{lb/s})\frac{1\,\text{hp}}{550\,\text{ft}\cdot\text{lb/s}} = 2.91\,\text{hp}}$$

Sample Problem 13.5



- In the second case, both bodies are accelerating. Apply Newton’s second law to each body to determine the required motor cable force.

$$a_D = 2.5\,\mathrm{ft/s^2}\uparrow \qquad a_C = -\tfrac{1}{2}a_D = 1.25\,\mathrm{ft/s^2}\downarrow$$

Free-body C:

$$+\downarrow \sum F_y = m_C a_C: \quad 800 - 2T = \frac{800}{32.2}(1.25) \qquad T = 384.5\,\mathrm{lb}$$

Free-body D:

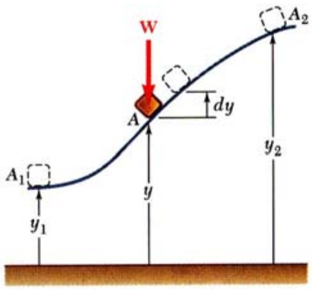
$$+\uparrow \sum F_y = m_D a_D: \quad F + T - 600 = \frac{600}{32.2}(2.5)$$

$$F + 384.5 - 600 = 46.6 \qquad F = 262.1\,\mathrm{lb}$$

$$Power = Fv_D = (262.1\,\mathrm{lb})(8\,\mathrm{ft/s}) = 2097\,\mathrm{ft\cdot lb/s}$$

$$\boxed{Power = (2097\,\mathrm{ft\cdot lb/s})\frac{1\,\mathrm{hp}}{550\,\mathrm{ft\cdot lb/s}} = 3.81\,\mathrm{hp}}$$

Potential Energy



- Work of the force of gravity $\vec{W}$,

  $U_{1\to 2} = W\,y_1 - W\,y_2$

- Work is independent of path followed; depends only on the initial and final values of *Wy*.

  $V_g = Wy$

  $=$ *potential energy* of the body with respect to *force of gravity*.

  $U_{1\to 2} = (V_g)_1 - (V_g)_2$

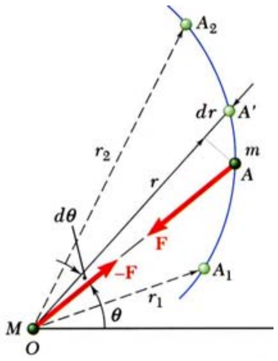
- Choice of datum from which the elevation *y* is measured is arbitrary.

- Units of work and potential energy are the same:

  $V_g = Wy = \mathrm{N\cdot m} = \mathrm{J}$

Vector Mechanics for Engineers: Dynamics

Potential Energy



- Previous expression for potential energy of a body with respect to gravity is only valid when the weight of the body can be assumed constant.
- For a space vehicle, the variation of the force of gravity with distance from the center of the earth should be considered.
- Work of a gravitational force,

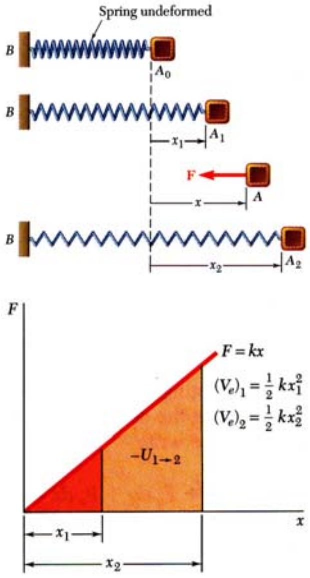
$$U_{1\to 2} = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

- Potential energy V_g when the variation in the force of gravity can not be neglected,

$$V_g = -\frac{GMm}{r} = -\frac{WR^2}{r}$$

Vector Mechanics for Engineers: Dynamics

Potential Energy



- Work of the force exerted by a spring depends only on the initial and final deflections of the spring,

$$U_{1\to 2} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

- The potential energy of the body with respect to the elastic force,

$$V_e = \tfrac{1}{2}kx^2$$
$$U_{1\to 2} = (V_e)_1 - (V_e)_2$$

- Note that the preceding expression for V_e is valid only if the deflection of the spring is measured from its undeformed position.

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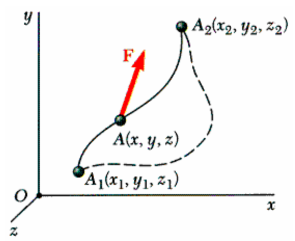
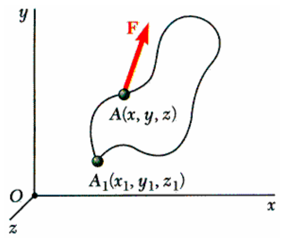
Seventh Edition

Conservative Forces





- Concept of potential energy can be applied if the work of the force is independent of the path followed by its point of application.

  $$U_{1\to2} = V(x_1, y_1, z_1) - V(x_2, y_2, z_2)$$

  Such forces are described as *conservative forces*.

- For any conservative force applied on a closed path,

  $$\oint \vec{F} \bullet d\vec{r} = 0$$

- Elementary work corresponding to displacement between two neighboring points,

  $$dU = V(x, y, z) - V(x + dx, y + dy, z + dz)$$
  $$= -dV(x, y, z)$$

  $$F_x dx + F_y dy + F_z dz = -\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)$$

  $$\vec{F} = -\left(\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\right) = -\mathbf{grad}\,V$$

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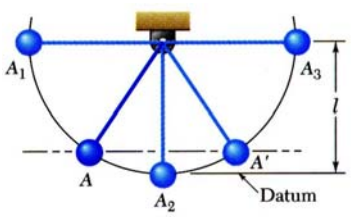
Seventh Edition

Conservation of Energy



$$T_1 = 0 \quad V_1 = W\ell$$
$$T_1 + V_1 = W\ell$$

$$T_2 = \tfrac{1}{2}mv_2^2 = \frac{1}{2}\frac{W}{g}(2g\ell) = W\ell \quad V_2 = 0$$
$$T_2 + V_2 = W\ell$$

- Work of a conservative force,

  $$U_{1\to2} = V_1 - V_2$$

- Concept of work and energy,

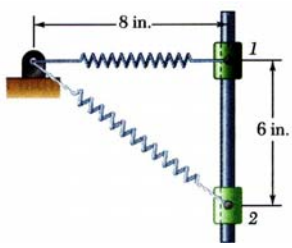
  $$U_{1\to2} = T_2 - T_1$$

- Follows that

  $$T_1 + V_1 = T_2 + V_2$$
  $$E = T + V = \text{constant}$$

- When a particle moves under the action of conservative forces, the total mechanical energy is constant.
- Friction forces are not conservative. Total mechanical energy of a system involving friction decreases.
- Mechanical energy is dissipated by friction into thermal energy. Total energy is constant.

Sample Problem 13.6



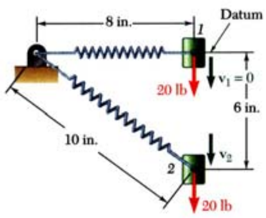
A 20 lb collar slides without friction along a vertical rod as shown. The spring attached to the collar has an undeflected length of 4 in. and a constant of 3 lb/in.

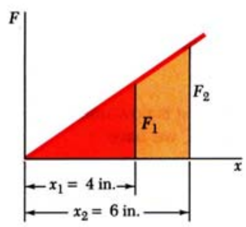
If the collar is released from rest at position 1, determine its velocity after it has moved 6 in. to position 2.

SOLUTION:

- Apply the principle of conservation of energy between positions 1 and 2.
- The elastic and gravitational potential energies at 1 and 2 are evaluated from the given information. The initial kinetic energy is zero.
- Solve for the kinetic energy and velocity at 2.

Sample Problem 13.6





SOLUTION:

- Apply the principle of conservation of energy between positions 1 and 2.

Position 1: $V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(3\,\text{lb/in.})(8\,\text{in.} - 4\,\text{in.})^2 = 24\,\text{in.}\cdot\text{lb}$

$$V_1 = V_e + V_g = 24\,\text{in.}\cdot\text{lb} + 0 = 2\,\text{ft}\cdot\text{lb}$$

$$T_1 = 0$$

Position 2: $V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(3\,\text{lb/in.})(10\,\text{in.} - 4\,\text{in.})^2 = 54\,\text{in.}\cdot\text{lb}$

$$V_g = Wy = (20\,\text{lb})(-6\,\text{in.}) = -120\,\text{in.}\cdot\text{lb}$$

$$V_2 = V_e + V_g = 54 - 120 = -66\,\text{in.}\cdot\text{lb} = -5.5\,\text{ft}\cdot\text{lb}$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\frac{20}{32.2}v_2^2 = 0.311v_2^2$$

Conservation of Energy:

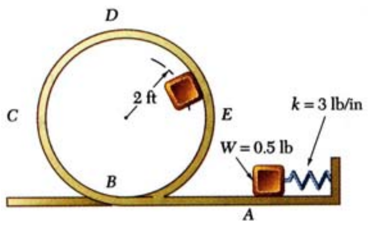
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\,\text{ft}\cdot\text{lb} = 0.311v_2^2 - 5.5\,\text{ft}\cdot\text{lb}$$

$$\boxed{v_2 = 4.91\,\text{ft/s}\downarrow}$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 13.7



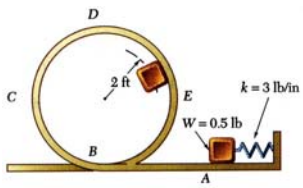
The 0.5 lb pellet is pushed against the spring and released from rest at A. Neglecting friction, determine the smallest deflection of the spring for which the pellet will travel around the loop and remain in contact with the loop at all times.

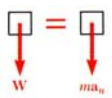
SOLUTION:

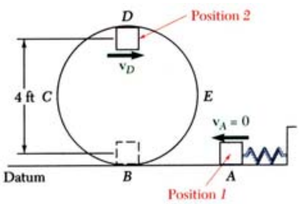
- Since the pellet must remain in contact with the loop, the force exerted on the pellet must be greater than or equal to zero. Setting the force exerted by the loop to zero, solve for the minimum velocity at D.
- Apply the principle of conservation of energy between points A and D. Solve for the spring deflection required to produce the required velocity and kinetic energy at D.

Vector Mechanics for Engineers: Dynamics

Sample Problem 13.7







SOLUTION:

- Setting the force exerted by the loop to zero, solve for the minimum velocity at D.

$$+\downarrow \sum F_n = ma_n: \quad W = ma_n \qquad mg = mv_D^2/r$$

$$v_D^2 = rg = (2\text{ ft})(32.2\text{ ft/s}) = 64.4\text{ ft}^2/\text{s}^2$$

- Apply the principle of conservation of energy between points A and D.

$$V_1 = V_e + V_g = \tfrac{1}{2}kx^2 + 0 = \tfrac{1}{2}(36\text{ lb/ft})x^2 = 18x^2$$

$$T_1 = 0$$

$$V_2 = V_e + V_g = 0 + Wy = (0.5\text{ lb})(4\text{ ft}) = 2\text{ ft}\cdot\text{lb}$$

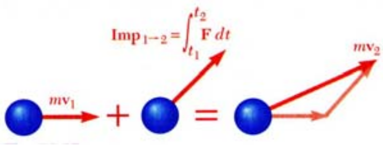
$$T_2 = \tfrac{1}{2}mv_D^2 = \frac{1}{2}\frac{0.5\text{ lb}}{32.2\text{ ft/s}^2}\left(64.4\text{ ft}^2/\text{s}^2\right) = 0.5\text{ ft}\cdot\text{lb}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 18x^2 = 0.5 + 2 \qquad \boxed{x = 0.3727\text{ ft} = 4.47\text{ in.}}$$

Vector Mechanics for Engineers: Dynamics

Principle of Impulse and Momentum



- Dimensions of the impulse of a force are
  *force*time*.
- Units for the impulse of a force are
  $\mathrm{N\cdot s} = \left(\mathrm{kg\cdot m/s^2}\right)\cdot\mathrm{s} = \mathrm{kg\cdot m/s}$

- From Newton's second law,

$$\vec{F} = \frac{d}{dt}(m\vec{v}) \qquad m\vec{v} = \text{linear momentum}$$

$$\vec{F}dt = d(m\vec{v})$$

$$\int_{t_1}^{t_2} \vec{F}dt = m\vec{v}_2 - m\vec{v}_1$$

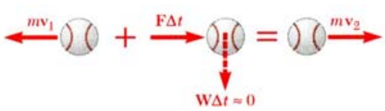
$$\int_{t_1}^{t_2} \vec{F}dt = \mathbf{Imp}_{1\to2} = \text{impulse of the force } \vec{F}$$

$$m\vec{v}_1 + \mathbf{Imp}_{1\to2} = m\vec{v}_2$$

- The final momentum of the particle can be obtained by adding vectorially its initial momentum and the impulse of the force during the time interval.

Vector Mechanics for Engineers: Dynamics

Impulsive Motion



- Force acting on a particle during a very short time interval that is large enough to cause a significant change in momentum is called an *impulsive force*.
- When impulsive forces act on a particle,
  $m\vec{v}_1 + \sum \vec{F}\,\Delta t = m\vec{v}_2$
- When a baseball is struck by a bat, contact occurs over a short time interval but force is large enough to change sense of ball motion.
- *Nonimpulsive forces* are forces for which $\vec{F}\Delta t$ is small and therefore, may be neglected.

Sample Problem 13.10

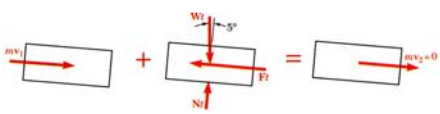
An automobile weighing 4000 lb is driven down a 5° incline at a speed of 60 mi/h when the brakes are applied, causing a constant total braking force of 1500 lb.

Determine the time required for the automobile to come to a stop.

SOLUTION:

- Apply the principle of impulse and momentum. The impulse is equal to the product of the constant forces and the time interval.

Sample Problem 13.10



SOLUTION:

- Apply the principle of impulse and momentum.

$$m\vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = m\vec{v}_2$$

Taking components parallel to the incline,

$$mv_1 + (W \sin 5°)t - Ft = 0$$

$$\left(\frac{4000}{32.2}\right)(88\,\text{ft/s}) + (4000 \sin 5°)t - 1500t = 0$$

$$\boxed{t = 9.49\,\text{s}}$$

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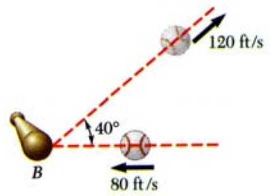
Sample Problem 13.11



A 4 oz baseball is pitched with a velocity of 80 ft/s. After the ball is hit by the bat, it has a velocity of 120 ft/s in the direction shown. If the bat and ball are in contact for 0.015 s, determine the average impulsive force exerted on the ball during the impact.

SOLUTION:

- Apply the principle of impulse and momentum in terms of horizontal and vertical component equations.

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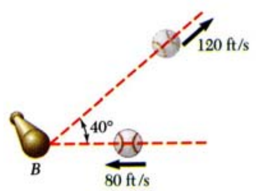
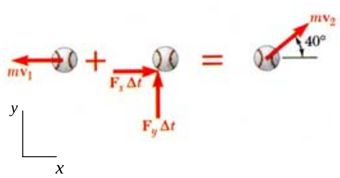
Sample Problem 13.11





SOLUTION:

- Apply the principle of impulse and momentum in terms of horizontal and vertical component equations.

$$m\vec{v}_1 + \mathbf{Imp}_{1\to 2} = m\vec{v}_2$$

x component equation:

$$-mv_1 + F_x\Delta t = mv_2\cos 40°$$

$$-\frac{4/16}{32.2}(80) + F_x(0.15) = \frac{4/16}{32.2}(120\cos 40°)$$

$$F_x = 89\,\text{lb}$$

y component equation:

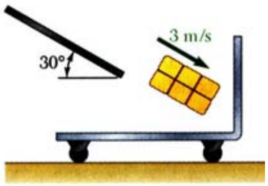
$$0 + F_y\Delta t = mv_2\sin 40°$$

$$F_y(0.15) = \frac{4/16}{32.2}(120\cos 40°)$$

$$F_y = 39.9\,\text{lb}$$

$$\boxed{\vec{F} = (89\,\text{lb})\vec{i} + (39.9\,\text{lb})\vec{j}, \quad F = 97.5\,\text{lb}}$$

Sample Problem 13.12



A 10 kg package drops from a chute into a 24 kg cart with a velocity of 3 m/s. Knowing that the cart is initially at rest and can roll freely, determine *(a)* the final velocity of the cart, *(b)* the impulse exerted by the cart on the package, and *(c)* the fraction of the initial energy lost in the impact.

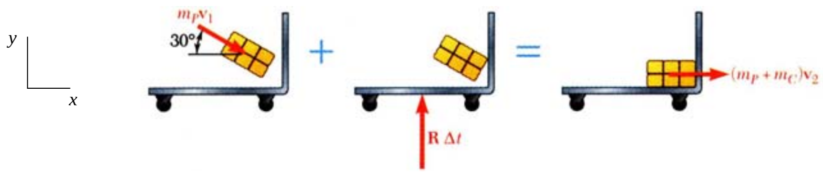
SOLUTION:

- Apply the principle of impulse and momentum to the package-cart system to determine the final velocity.
- Apply the same principle to the package alone to determine the impulse exerted on it from the change in its momentum.

Sample Problem 13.12

SOLUTION:

- Apply the principle of impulse and momentum to the package-cart system to determine the final velocity.



$$m_p \vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = (m_p + m_c)\vec{v}_2$$

x components:

$$m_p v_1 \cos 30° + 0 = (m_p + m_c) v_2$$

$$(10\text{ kg})(3\text{ m/s})\cos 30° = (10\text{ kg} + 25\text{ kg})v_2$$

$$\boxed{v_2 = 0.742\text{ m/s}}$$

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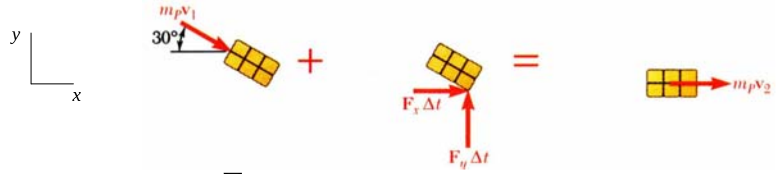
Sample Problem 13.12

- Apply the same principle to the package alone to determine the impulse exerted on it from the change in its momentum.



$$m_p\vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = m_p\vec{v}_2$$

x components:

$$m_p v_1\cos 30° + F_x\Delta t = m_p v_2$$

$$(10\text{ kg})(3\text{ m/s})\cos 30° + F_x\Delta t = (10\text{ kg})v_2 \qquad F_x\Delta t = -18.56\text{ N}\cdot\text{s}$$

y components:

$$-m_p v_1\sin 30° + F_y\Delta t = 0$$

$$-(10\text{ kg})(3\text{ m/s})\sin 30° + F_y\Delta t = 0 \qquad F_y\Delta t = 15\text{ N}\cdot\text{s}$$

$$\boxed{\sum \mathbf{Imp}_{1\to 2} = \vec{F}\Delta t = (-18.56\text{ N}\cdot\text{s})\vec{i} + (15\text{ N}\cdot\text{s})\vec{j} \qquad F\Delta t = 23.9\text{ N}\cdot\text{s}}$$

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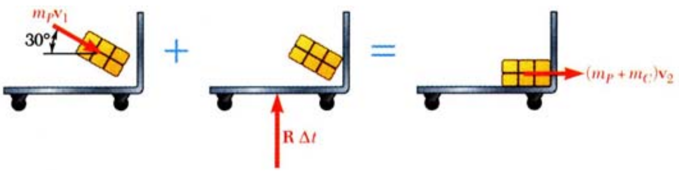
Sample Problem 13.12



To determine the fraction of energy lost,

$$T_1 = \tfrac{1}{2}m_p v_1^2 = \tfrac{1}{2}(10\text{ kg})(3\text{ m/s})^2 = 45\text{ J}$$

$$T_1 = \tfrac{1}{2}(m_p + m_c)v_2^2 = \tfrac{1}{2}(10\text{ kg} + 25\text{ kg})(0.742\text{ m/s})^2 = 9.63\text{ J}$$

$$\boxed{\frac{T_1 - T_2}{T_1} = \frac{45\text{ J} - 9.63\text{ J}}{45\text{ J}} = 0.786}$$

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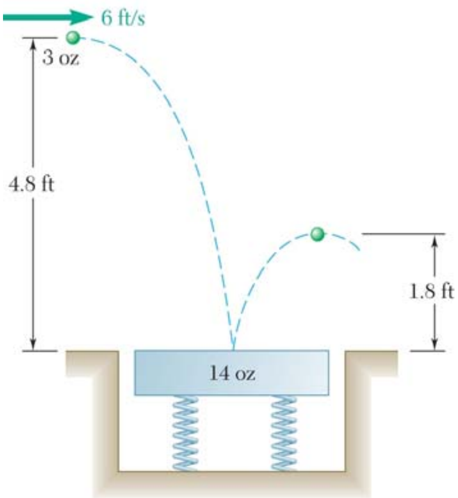
Problem 13.150



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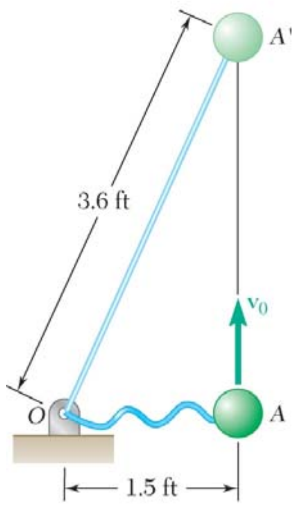
Problem 13.154



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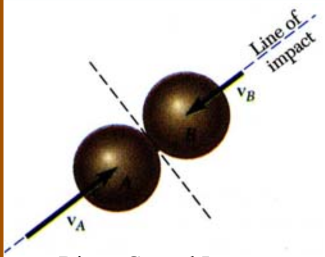
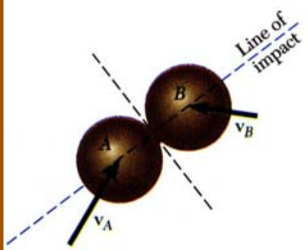
Impact



Direct Central Impact



Oblique Central Impact

- *Impact:* Collision between two bodies which occurs during a small time interval and during which the bodies exert large forces on each other.
- *Line of Impact:* Common normal to the surfaces in contact during impact.
- *Central Impact:* Impact for which the mass centers of the two bodies lie on the line of impact; otherwise, it is an *eccentric impact.*.
- *Direct Impact:* Impact for which the velocities of the two bodies are directed along the line of impact.
- *Oblique Impact:* Impact for which one or both of the bodies move along a line other than the line of impact.

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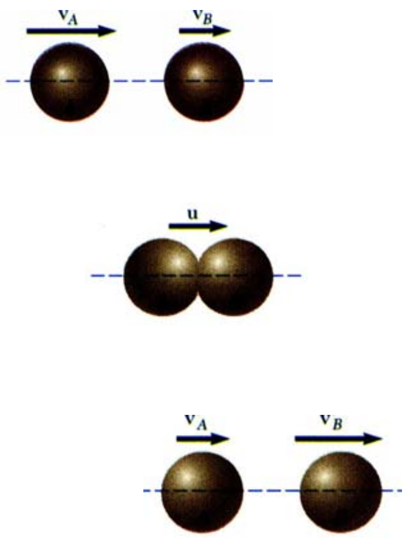
Direct Central Impact



- Bodies moving in the same straight line, $v_A > v_B$.
- Upon impact the bodies undergo a *period of deformation*, at the end of which, they are in contact and moving at a common velocity.
- A *period of restitution* follows during which the bodies either regain their original shape or remain permanently deformed.
- Wish to determine the final velocities of the two bodies. The total momentum of the two body system is preserved,

$$m_A v_A + m_B v_B = m_B v'_B + m_B v'_B$$

- A second relation between the final velocities is required.

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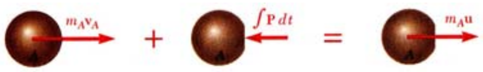
Direct Central Impact



- Period of deformation: $m_A v_A - \int P dt = m_A u$

- Period of restitution: $m_A u - \int R dt = m_A v'_A$

$$e = \text{coefficient of restitution} = \frac{\int R dt}{\int P dt} = \frac{u - v'_A}{v_A - u}$$

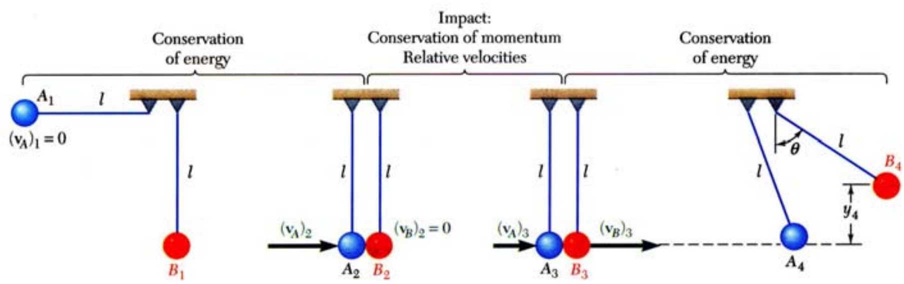
$$0 \le e \le 1$$

- A similar analysis of particle B yields $\quad e = \dfrac{v'_B - u}{u - v_B}$
- Combining the relations leads to the desired second relation between the final velocities. $\quad v'_B - v'_A = e(v_A - v_B)$
- *Perfectly plastic impact*, $e = 0$: $\quad v'_B = v'_A = v' \quad m_A v_A + m_B v_B = (m_A + m_B)v'$
- *Perfectly elastic impact*, $e = 1$: Total energy and total momentum conserved. $\quad v'_B - v'_A = v_A - v_B$

Vector Mechanics for Engineers: Dynamics

Problems Involving Energy and Momentum

- Three methods for the analysis of kinetics problems:
  - Direct application of Newton's second law
  - Method of work and energy
  - Method of impulse and momentum
- Select the method best suited for the problem or part of a problem under consideration.



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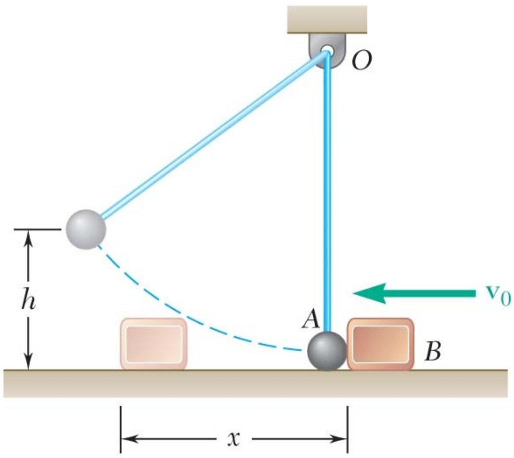
Problem 13.176



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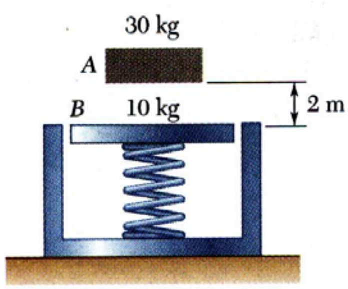
Sample Problem 13.17



A 30 kg block is dropped from a height of 2 m onto the the 10 kg pan of a spring scale. Assuming the impact to be perfectly plastic, determine the maximum deflection of the pan. The constant of the spring is $k = 20$ kN/m.

SOLUTION:

- Apply the principle of conservation of energy to determine the velocity of the block at the instant of impact.
- Since the impact is perfectly plastic, the block and pan move together at the same velocity after impact. Determine that velocity from the requirement that the total momentum of the block and pan is conserved.
- Apply the principle of conservation of energy to determine the maximum deflection of the spring.

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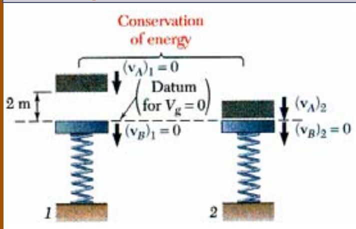
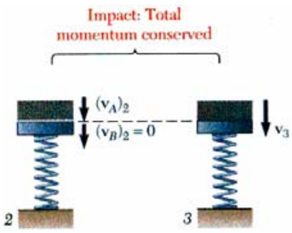
Sample Problem 13.17





SOLUTION:

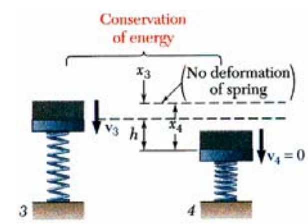
- Apply principle of conservation of energy to determine velocity of the block at instant of impact.

$$T_1 = 0 \qquad V_1 = W_A y = (30)(9.81)(2) = 588\ \text{J}$$
$$T_2 = \tfrac{1}{2} m_A (v_A)_2^2 = \tfrac{1}{2}(30)(v_A)_2^2 \qquad V_2 = 0$$
$$T_1 + V_1 = T_2 + V_2$$
$$0 + 588\ \text{J} = \tfrac{1}{2}(30)(v_A)_2^2 + 0 \qquad (v_A)_2 = 6.26\ \text{m/s}$$

- Determine velocity after impact from requirement that total momentum of the block and pan is conserved.

$$m_A (v_A)_2 + m_B (v_B)_2 = (m_A + m_B) v_3$$
$$(30)(6.26) + 0 = (30+10) v_3 \qquad v_3 = 4.70\ \text{m/s}$$

Sample Problem 13.17



Initial spring deflection due to pan weight:

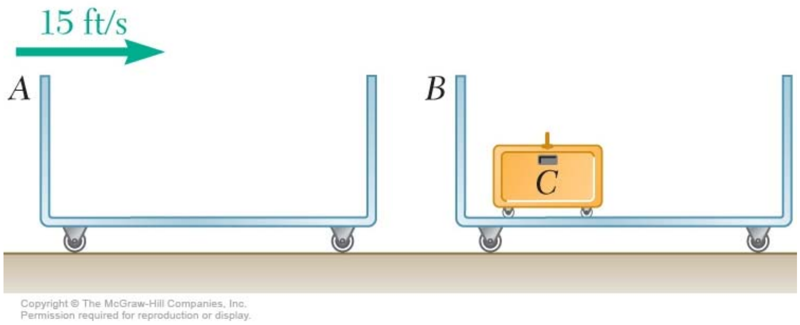
$$x_3 = \frac{W_B}{k} = \frac{(10)(9.81)}{20 \times 10^3} = 4.91 \times 10^{-3}\ \text{m}$$

- Apply the principle of conservation of energy to determine the maximum deflection of the spring.

$$T_3 = \tfrac{1}{2}(m_A + m_B) v_3^2 = \tfrac{1}{2}(30+10)(4.7)^2 = 442\ \text{J}$$
$$V_3 = V_g + V_e$$
$$= 0 + \tfrac{1}{2} k x_3^2 = \tfrac{1}{2}\left(20 \times 10^3\right)\left(4.91 \times 10^{-3}\right)^2 = 0.241\ \text{J}$$
$$T_4 = 0$$
$$V_4 = V_g + V_e = (W_A + W_B)(-h) + \tfrac{1}{2} k x_4^2$$
$$= -392(x_4 - x_3) + \tfrac{1}{2}\left(20 \times 10^3\right) x_4^2$$
$$= -392\left(x_4 - 4.91 \times 10^{-3}\right) + \tfrac{1}{2}\left(20 \times 10^3\right) x_4^2$$
$$T_3 + V_3 = T_4 + V_4$$
$$442 + 0.241 = 0 - 392\left(x_4 - 4.91 \times 10^{-3}\right) + \tfrac{1}{2}\left(20 \times 10^3\right) x_4^2$$
$$x_4 = 0.230\ \text{m}$$

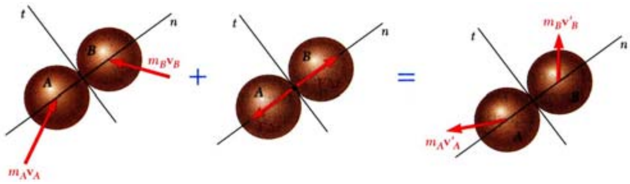
$$h = x_4 - x_3 = 0.230\ \text{m} - 4.91 \times 10^{-3}\ \text{m} \qquad \boxed{h = 0.225\ \text{m}}$$

Problem 13.177



Vector Mechanics for Engineers: Dynamics

Oblique Central Impact (freely moving particles)



- Final velocities are unknown in magnitude and direction. Four equations are required.

- No tangential impulse component; tangential component of momentum for each particle is conserved.
  $$(v_A)_t = (v'_A)_t \qquad (v_B)_t = (v'_B)_t$$
- Normal component of total momentum of the two particles is conserved.
  $$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$
- Normal components of relative velocities before and after impact are related by the coefficient of restitution.
  $$(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]$$

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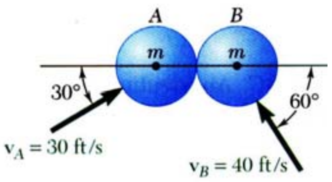
Sample Problem 13.15



The magnitude and direction of the velocities of two identical frictionless balls before they strike each other are as shown. Assuming $e = 0.9$, determine the magnitude and direction of the velocity of each ball after the impact.

SOLUTION:

- Resolve the ball velocities into components normal and tangential to the contact plane.
- Tangential component of momentum for each ball is conserved.
- Total normal component of the momentum of the two ball system is conserved.
- The normal relative velocities of the balls are related by the coefficient of restitution.
- Solve the last two equations simultaneously for the normal velocities of the balls after the impact.

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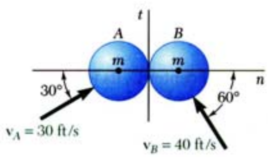
Vector Mechanics for Engineers: Dynamics

Sample Problem 13.15

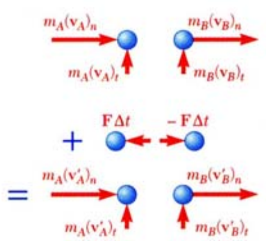


SOLUTION:

- Resolve the ball velocities into components normal and tangential to the contact plane.

$$(v_A)_n = v_A \cos 30° = 26.0\,\text{ft/s} \qquad (v_A)_t = v_A \sin 30° = 15.0\,\text{ft/s}$$

$$(v_B)_n = -v_B \cos 60° = -20.0\,\text{ft/s} \qquad (v_B)_t = v_B \sin 60° = 34.6\,\text{ft/s}$$



- Tangential component of momentum for each ball is conserved.

$$(v'_A)_t = (v_A)_t = 15.0\,\text{ft/s} \qquad (v'_B)_t = (v_B)_t = 34.6\,\text{ft/s}$$

- Total normal component of the momentum of the two ball system is conserved.

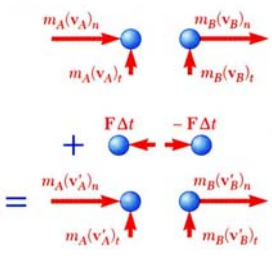
$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$m(26.0) + m(-20.0) = m(v'_A)_n + m(v'_B)_n$$

$$(v'_A)_n + (v'_B)_n = 6.0$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 13.15

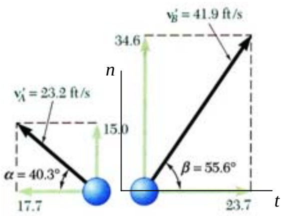


- The normal relative velocities of the balls are related by the coefficient of restitution.

$$(v'_A)_n - (v'_B)_n = e\left[(v_A)_n - (v_B)_n\right]$$
$$= 0.90[26.0 - (-20.0)] = 41.4$$

- Solve the last two equations simultaneously for the normal velocities of the balls after the impact.

$$(v'_A)_n = -17.7\,\text{ft/s} \qquad (v'_B)_n = 23.7\,\text{ft/s}$$



$$\vec{v}'_A = -17.7\vec{\lambda}_t + 15.0\vec{\lambda}_n$$
$$v'_A = 23.2\,\text{ft/s} \quad \tan^{-1}\left(\frac{15.0}{17.7}\right) = 40.3°$$
$$\vec{v}'_B = 23.7\vec{\lambda}_t + 34.6\vec{\lambda}_n$$
$$v'_B = 41.9\,\text{ft/s} \quad \tan^{-1}\left(\frac{34.6}{23.7}\right) = 55.6°$$

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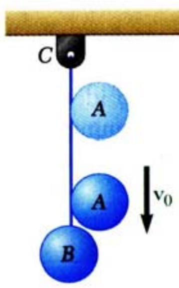
Sample Problem 13.16



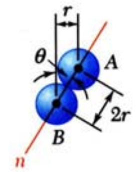
Ball *B* is hanging from an inextensible cord. An identical ball *A* is released from rest when it is just touching the cord and acquires a velocity v_0 before striking ball *B*. Assuming perfectly elastic impact ($e = 1$) and no friction, determine the velocity of each ball immediately after impact.

SOLUTION:

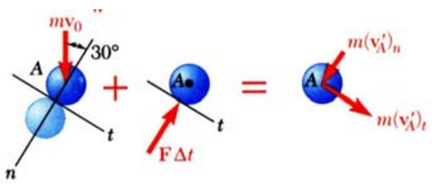
- Determine orientation of impact line of action.
- The momentum component of ball *A* tangential to the contact plane is conserved.
- The total horizontal momentum of the two ball system is conserved.
- The relative velocities along the line of action before and after the impact are related by the coefficient of restitution.
- Solve the last two expressions for the velocity of ball *A* along the line of action and the velocity of ball *B* which is horizontal.

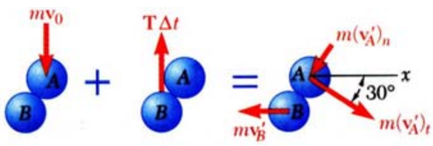
Vector Mechanics for Engineers: Dynamics

Sample Problem 13.16



$$\sin\theta = \frac{r}{2r} = 0.5$$
$$\theta = 30^\circ$$





SOLUTION:

- Determine orientation of impact line of action.
- The momentum component of ball *A* tangential to the contact plane is conserved.

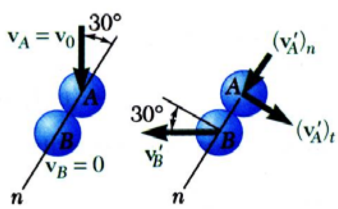
$$m\vec{v}_A + \vec{F}\Delta t = m\vec{v}'_A$$
$$mv_0 \sin 30^\circ + 0 = m(v'_A)_t$$
$$(v'_A)_t = 0.5v_0$$

- The total horizontal (*x* component) momentum of the two ball system is conserved.

$$m\vec{v}_A + \vec{T}\Delta t = m\vec{v}'_A + m\vec{v}'_B$$
$$0 = m(v'_A)_t \cos 30^\circ - m(v'_A)_n \sin 30^\circ - mv'_B$$
$$0 = (0.5v_0)\cos 30^\circ - (v'_A)_n \sin 30^\circ - v'_B$$
$$0.5(v'_A)_n + v'_B = 0.433v_0$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 13.16

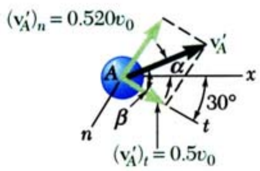


- The relative velocities along the line of action before and after the impact are related by the coefficient of restitution.

$$(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]$$
$$v'_B \sin 30^\circ - (v'_A)_n = v_0 \cos 30^\circ - 0$$
$$0.5v'_B - (v'_A)_n = 0.866v_0$$

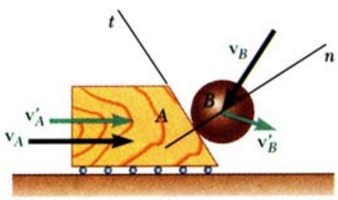
- Solve the last two expressions for the velocity of ball *A* along the line of action and the velocity of ball *B* which is horizontal.

$$(v'_A)_n = -0.520v_0 \qquad v'_B = 0.693v_0$$



$$\boxed{\begin{array}{l} \vec{v}'_A = 0.5v_0\vec{\lambda}_t - 0.520v_0\vec{\lambda}_n \\ v'_A = 0.721v_0 \quad \beta = \tan^{-1}\left(\frac{0.52}{0.5}\right) = 46.1^\circ \\ \alpha = 46.1^\circ - 30^\circ = 16.1^\circ \\ v'_B = 0.693v_0 \leftarrow \end{array}}$$

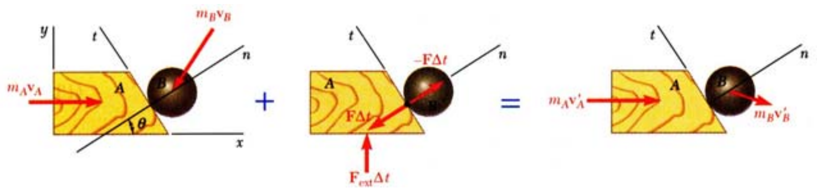
Oblique Central Impact (one particle is constrained in its motion)



- Block constrained to move along horizontal surface.
- Impulses from internal forces $\vec{F}$ and $-\vec{F}$ along the n axis and from external force $\vec{F}_{ext}$ exerted by horizontal surface and directed along the vertical to the surface.
- Final velocity of ball unknown in direction and magnitude and unknown final block velocity magnitude. Three equations required.

Vector Mechanics for Engineers: Dynamics

Oblique Central Impact



- Tangential momentum of ball is conserved.

$$(v_B)_t = (v'_B)_t$$

- Total horizontal momentum of block and ball is conserved.

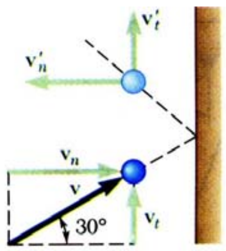
$$m_A(v_A) + m_B(v_B)_x = m_A(v'_A) + m_B(v'_B)_x$$

- Normal component of relative velocities of block and ball are related by coefficient of restitution.

$$(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]$$

- Note: Validity of last expression does not follow from previous relation for the coefficient of restitution. A similar but separate derivation is required.

Sample Problem 13.14(impact of a particle with a massive rigid body)

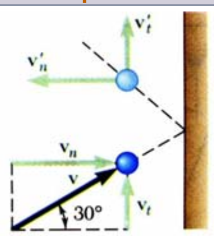


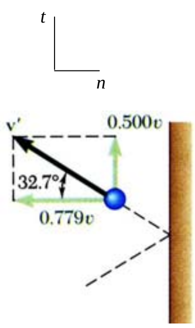
A ball is thrown against a frictionless, vertical wall. Immediately before the ball strikes the wall, its velocity has a magnitude v and forms angle of 30º with the horizontal. Knowing that $e = 0.90$, determine the magnitude and direction of the velocity of the ball as it rebounds from the wall.

SOLUTION:

- Resolve ball velocity into components normal and tangential to wall.
- Impulse exerted by the wall is normal to the wall. Component of ball momentum tangential to wall is conserved.
- Assume that the wall has infinite mass so that wall velocity before and after impact is zero. Apply coefficient of restitution relation to find change in normal relative velocity between wall and ball, i.e., the normal ball velocity.

Sample Problem 13.14





SOLUTION:

- Resolve ball velocity into components parallel and perpendicular to wall.

$$v_n = v\cos 30° = 0.866v \qquad v_t = v\sin 30° = 0.500v$$

- Component of ball momentum tangential to wall is conserved.

$$v'_t = v_t = 0.500v$$

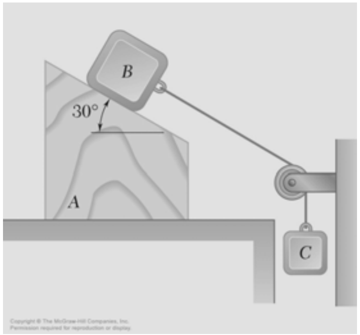
- Apply coefficient of restitution relation with zero wall velocity.

$$0 - v'_n = e(v_n - 0)$$
$$v'_n = -0.9(0.866v) = -0.779v$$

$$\boxed{\begin{array}{l}\vec{v}' = -0.779v\,\vec{\lambda}_n + 0.500v\,\vec{\lambda}_t \\ v' = 0.926v \quad \tan^{-1}\left(\dfrac{0.779}{0.500}\right) = 32.7°\end{array}}$$

Vector Mechanics for Engineers: Dynamics

Quiz 2



Block *B* having a mass of 9 kg is initially at rest as shown on the upper surface of a 22.5 kg wedge *A* which is supported by a horizontal surface. A 2 kg block *C* is connected to block *B* by a cord, which passes over a pulley of negligible mass. Using computational software and denoting by the coefficient of friction at all surfaces, calculate the initial acceleration of the wedge and the initial acceleration of block *B* relative to the wedge for values of μ≥0. Use 0.01 increments for until the wedge does not move and then use 0.1 increments until no motion occurs.

Vector Mechanics for Engineers: Dynamics

```
%% Problem 12.C1 and Quiz 2
clear all, clc, fprintf('\n\n\t   Solution of Problem 12.C1 and Quiz 2 ')
g = 9.81;

for me242=1:2

   if me242==1
      Wa = 200; % homework
      Wb = 80;  % homework
      Wc = 18;  % homework
      fprintf('\n\n\t\t\t\t----Problem 12.C1----\n\n')
   elseif me242==2
      Wa = 22.5*g; % quiz2
      Wb = 9*g;    % quiz2
      Wc = 2*g;    % quiz2
      fprintf('\n\n\t\t\t\t----Quiz 2----\n\n')
   end
   ma = Wa/g;
   mb = Wb/g;
   mc = Wc/g;

   t = 30;
   th = t*pi/180;
   Mu = 0;
   A = (1-Mu.^2)*sin(th)-2*Mu*cos(th);
   a_A = g*(A*Wb*cos(th)-Wa*Mu)/(Wa+Wb*A*sin(th));
   % print heading
   fprintf('  Mu      Accel. of A (m/s^2)     Accel. of B wrt A, (m/s^2)\n');
```

```
while a_A > 0
    a_BwA = (1/(Wb+Wc))*(g*(Wc-Wb*(Mu*cos(th)-sin(th)))+a_A*(Wb*Mu*sin(th)+(Wc+Wb)*cos(th)));
    fprintf('  %3.2f        %4.3f                %4.3f\n',Mu,a_A,a_BwA);
    Mu = Mu+0.01;
    A = (1-Mu.^2)*sin(th)-2*Mu*cos(th);
    a_A = g*(A*Wb*cos(th)-Wa*Mu)/(Wa+Wb*A*sin(th));
  end
  % Increase Mu to the next tenth
  Mu = 0.20;
  a_BwA = (g/(Wb+Wc))*(Wc-Wb*(Mu*cos(th)-sin(th)));
  % print heading
  fprintf('\n');
  fprintf('  Mu      Accel. of B wrt A, (m/s^2)\n');
  while a_BwA > 0
    a_BwA = (g/(Wb+Wc))*(Wc-Wb*(Mu*cos(th)-sin(th)));
    if a_BwA > 0
      fprintf('  %3.2f          %4.3f\n',Mu,a_BwA);
    end
    Mu = Mu+0.10;
  end


end
```