Nonlinear Pricing under Inequity Aversion*

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Abstract

We consider a monopoly which practices nonlinear pricing, where buyers may have inequity-averse preferences. Each buyer has a valuation for the good, drawn from a distribution. Monopoly knows the distribution but not the realizations. We introduce the possibility that any buyer can be inequity-averse (fair types) or not (neutral types). Fair types get a disutility from inequity and this is captured through a utility function in the spirit of the one introduced by Fehr and Schmidt (1999). We characterize the optimal nonlinear pricing and show that the degree of suboptimality increases, and the monopoly profit goes down, as the degree of the inequity aversion increases. We also show that some or all of the neutral types with low demand are strictly better off, when there are inequity-averse types in the environment.

Keywords : price discrimination, nonlinear pricing, fairness, inequity aversion.

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1 Introduction

Pricing strategies of a monopoly facing different demand groups have been widely studied. Depending on how much a monopoly can tell different buyers apart in terms of their values of willingness to pay, the monopoly can adopt different degrees of price discrimination. When the monopoly knows that there are different groups of buyers with different demand structures, but is unable to tell which buyer is in which group, the monopoly can practice second degree price discrimination. This could be done through two-part tariffs, which involve a fixed fee and a unit price, or more generally through a nonlinear pricing scheme. There is a large number of real-life examples where the firms are engaging in price discrimination practices even when they do not directly observe which demand group a particular buyer belongs to. For instance, different memberships for gyms or clubs (some with a higher fee and better facility access, some with lower fee and limited facility access), bulk discounts (in form of dropping average price as quantity purchased goes up, for instance, in electricity consumption), tying and bundling (conditioning the sale of one good on the purchase of the other good from the same seller, for instance, tying printers and cartridges) and different pricing by airline companies (in form of different prices for the same flight that vary with the timing of the purchase or with the buyer's location at the time of purchase). The purpose of these practices is to get different buyers to buy the good at prices closer to their own individual willingness to pay values. If the firm manages to come with such a pricing, it receives a higher profit.

When a monopoly applies nonlinear pricing, it offers a set of pairs of total quantity and total payment, and lets each buyer choose whichever pair she wants. The pairs are determined so that each pair is targeted at a specific demand group, and the buyers in that demand group find it optimal to get the pair, which is targeted at them.[1] In a simple setting with two types, what happens is that the monopoly leaves no surplus to the low demand type buyers, and leaves positive surplus to the high demand type buyers, as information rents. Moreover, the low demand type ends up consuming a quantity that is socially suboptimal, more precisely, the marginal benefit from its consumption is higher than its marginal cost.[2]

When there are different types of agents in an asymmetric information environment where the firm cannot tell which buyer is of which type, these different types of buyers are likely to end up with heterogeneous allocations with possibly different average prices, which may yield to different information rents. For instance, a low demand type may end up with no information rent (zero net surplus), whereas a high demand type may receive a positive information rent (positive net surplus), and that may make the low demand type

[1] See Goldman, Leland, and Sibley (1984), Maskin and Riley (1984) and Oren, Smith, and Wilson (1984) for relatively early studies which have focused on this problem.

[2] Tirole (1988) has depicted these main findings.

(and/or the high demand type) unhappy because of the inequity in terms of different net surpluses. Thus, it is plausible to be concerned about buyers' preferences for fairness or their potential aversion to inequity which may result from the differences in these information rents each buyer gets. It is, therefore, important to consider inequity-averse buyers, and to analyze the effects of such aversion on the incentive scheme the monopoly designs. Milgrom and Roberts (1992) pointed out that when providing incentives, preferences that take into account others' well-being do matter: "A given level of pay may be viewed as good or bad, acceptable or unacceptable, depending on the compensation of others in the reference group, and as such may result in different behavior." Also, Boyle (2000) indicated that the consumer reaction to price discrimination usually has a sense of unfairness: "...people often react to differential pricing for the same good with a sense of unfairness. No matter how many times they are lectured by the economists that it is actually to the benefit of all that producers be able to charge different prices to groups with different ability and willingness to pay, the popular reaction is normally 'that's not fair'."

There has been some influential studies, both theoretical and experimental, that explore preferences for fairness and inequity aversion. Prasnikar and Roth (1992) provided experimental results that point towards fairness concerns in various game experiments, and Rabin (1993) studied the notion of fairness from a game theoretical perspective. In their seminal paper, Fehr and Schmidt (1999) studied inequity-averse preferences and have provided a simple model of inequity aversion together with a utility form that represents preferences for fairness. In this paper, we incorporate fairness concerns into a model of a nonlinear pricing monopoly, adopting a version of the utility form provided in Fehr and Schmidt (1999) to account for inequity aversion. We consider inequity aversion in the sense that if a buyer receives a net surplus which is different from the net surpluses of other buyers, then this buyer may get disutility from those differences.

We consider both discrete and continuum types environment where a monopoly is using a nonlinear pricing scheme facing different types of buyers: different both in terms of their valuations for the good and also in terms of whether they have preferences for fairness or not. The monopoly knows that there are these different types but cannot tell which buyer is in which group. We first solve for the optimal nonlinear pricing assuming all types are served, and show that the degree of suboptimality strictly increases as the degree of inequity aversion increases. This follows from the extra distortion inequity aversion creates, which increases the net surplus of the low demand type, and consequently, more incentives are needed to be provided for the high demand type. This is provided through a smaller quantity for the low demand type in order to make that bundle less attractive for the high demand type. This lower quantity increases the difference between the marginal benefit and the marginal cost of the quantity targeted at the low demand type. Second, we show that, in the discrete case, the inequity-neutral type with a lower demand is strictly better off, and, in the continuum case, a positive mass of low demand inequity-neutral types are strictly better off, when there are inequity-averse types relative to the case when there are no inequity-averse types in the environment. The intuition behind this result is that leaving zero net surplus for the neutral low demand type is not compatible with the participation constraint of the inequity-averse low demand type, forcing the monopoly to leave positive net surplus for the neutral low demand type. We also show that the monopoly profit decreases as the degree of inequity aversion increases in both discrete and continuum cases. This is due to the distortion generated by inequity aversion, which makes it more costly to provide incentives to different type of buyers to self select. We also analyze various extensions and discuss some issues, including the case when the monopoly is allowed to exclude some of the buyers from the market by not selling to them, an alternative individual rationality condition, and also an alternative evaluation of the disutility from inequity.

The scope of this paper falls into the intersection of the literature on nonlinear pricing and the literature on inequity aversion. Nonlinear pricing literature can be thought of two strands, one on monopoly pricing and the other on oligopolistic competition. Since our focus is on the monopoly pricing, we will discuss the former one.[3] Varian (1989) provides an extensive discussion of the different types of price discrimination, including nonlinear pricing, and their applications. When a monopoly is considered, the optimal nonlinear pricing involves marginal cost pricing for the last unit, and higher marginal prices for all lower quantities (Mussa and Rosen, 1978).[4] As we discussed above, the monopoly leaves some information rent for the high demand type and no surplus for the low demand type, and the quantity for the low demand type is socially suboptimal (Tirole, 1988).[5] Varian (1985) explores the welfare implications of a third degree price discrimination and shows that the effect is positive only if the market output increases. Hendel, Lizzeri, and Roketskiy (2014) develop a model of nonlinear pricing of storable goods and show that storability restricts monopolist's ability to extract surplus. In terms of behavioral aspects, such as biased beliefs, time-inconsistency, ambiguity aversion and loss aversion, and their implications in the nonlinear pricing context, there is a growing number of studies. When consumers have biased prior beliefs regarding their future preferences, the optimal nonlinear pricing of a monopoly contains a risky offer as well as a safe offer that guarantees the consumer her reservation value in each state (Eliaz and Spiegler, 2008). In an optimal set of two-part tariffs problem of a firm which is facing time-inconsistent partially naive agents, an investment

[3]For the literature on nonlinear pricing with oligopolistic competition, see Stole (2007) for an extensive chapter on competition and price discrimination in the Handbook of Industrial Organization 3. Also, see Armstrong (2016) for an extensive survey and analysis on nonlinear pricing and bundling for multi-seller case (as well as the single-seller case), and for both discrete and continuum types of buyers.

[4]When buyer entry is introduced, through learning own preference type after incurring privately known entry costs, the optimal nonlinear pricing contract changes significantly: distortion, exclusion and bunching are reduced (Ye and Zhang, 2017).

[5]See Maskin and Riley (1984) and Wilson (1996) for a mechanism design approach for a monopoly's nonlinear pricing problem.

good (immediate cost) is priced below its marginal cost, whereas a leisure good (immediate benefit) is priced over its marginal cost (DellaVigna and Malmendier, 2004). When buyers are loss-averse, there may be larger downward distortions resulting from screening and efficiency gains relative to the second best pricing without loss aversion (Carbajal and Ely, 2016). Similarly, with loss-averse buyers, a monopoly finds it optimal to offer the same bundle (full pooling menu) if the likelihood of low demand buyer is sufficiently large and the loss aversion is in an intermediate range (Hahn, Kim, Kim, and Lee, 2016). When the monopoly has information ambiguity, the optimal nonlinear pricing involves bunching at the bottom, and the distortion is reduced (Zheng, Wang, and Li, 2015). Our findings under inequity aversion resemble some of these findings: we also obtain pooling at the bottom for the discrete case, where the inequity-averse and inequity-neutral low demand type buyers receive the same bundle, and also pooling (across neutral and fair types) in continuum type case.

Regarding the inequity-averse preferences, in their seminal paper, Fehr and Schmidt (1999) provide a simple representation of other-regarding preferences, and they solve the following puzzle: when players are given the opportunity to punish free riders, stable cooperation is maintained, although punishment is costly for those who punish. If some people care about equity, the puzzle is solved. Since their study, a large number of both theoretical and empirical studies emerged on other-regarding preferences, including inequity aversion and its implications.[6] In the contracting context, there is experimental evidence that shows that when fairness concerns are present, principals prefer less complete contracts over more complete ones, although the standard self-interest model predicts otherwise (Fehr and Schmidt, 2000). In a moral hazard setting, there is experimental evidence showing that bonus contracts are chosen over incentive contracts implying that principals may have fairness concerns (Fehr, Klein, and Schmidt, 2007).[7] In a principal-agent setup where the agent has inequity aversion, the optimal contract involves linear sharing rules (Englmaier and Wambach, 2010). Experimental evidence shows that there are efficiency gains through an increase in the set of enforceable actions, when both sides in the market have reciprocity (Fehr, Gächter, and Kirchsteiger, 1997). There is also experimental evidence for buyers setting prices sufficiently above the market clearing price when there are fairness concerns, and sellers responding with high quality levels (Fehr, Kirchsteiger, and Riedl, 1993).[8] In a related study, Englmaier, Gratz, and Reisinger (2012), consider third degree price discrimination when consumers have reciprocity, and show that the difference in prices is smaller relative

[6] Also see Fehr and Schmidt (2006) for empirical foundations and theoretical approaches of other-regarding preferences.

[7] However, trust contracts are not chosen as much as incentive contracts implying the agents are not necessarily inequity-averse.

[8] For a theoretical study on reciprocity that takes both consequences and underlying intentions into account, see Falk and Fischbacher (2006). Also see Dufwenberg and Kirchsteiger (2004) for an existence result for sequential reciprocity equilibrium, for dynamic games with reciprocity.

to the case with no reciprocity. Our study is similar to theirs in the sense that we both analyze the affect of inequity aversion on the price discrimination, however, we focus on second degree price discrimination whereas they focus on third degree price discrimination. Also, the way fairness enters the buyer's utility is different: they follow the reciprocity model provided by Falk and Fischbacher (2006), and we adopt the inequity aversion model provided by Fehr and Schmidt (1999). In our model, for a buyer inequity aversion results from the differences between her own surplus and other buyers' surpluses, whereas in their model a buyer reciprocates toward the seller, because of inequity between her and another buyer.

Given the volume of experimental studies that provide evidence for fairness concerns of agents and inequity-averse preferences, it is important to analyze implications of inequity aversion in various economic models and environments. In nonlinear pricing literature, the buyers are assumed to have only self-regarding preferences and no fairness concerns. In the light of the experimental evidence that suggests otherwise, it becomes crucial to adjust existing theoretical models to incorporate such preferences. This paper contributes to the literature in this sense, by studying the implications of inequity aversion in the context of monopoly's nonlinear pricing, where buyers may get disutility when their net surplus resulting from the nonlinear pricing adopted by the monopoly differs from the net surpluses of other buyers.[9]

We depict the model in Section 2. We consider a discrete type space and solve the optimal nonlinear pricing in Section 3. In Section 4, we solve the model with continuum set of types and characterize the optimal nonliner pricing and its efficiency properties. In Section 5, we provide an analysis for the case where it is allowed to exclude some buyers, and also discuss a few other issues. Section 6 concludes.

2 Model

There is a monopoly who produces and sells a single product at a constant marginal cost $c > 0$ to a set of buyers. The monopoly engages in nonlinear pricing by offering a set of quantity-total payment bundles. The monopoly faces a total demand, which consists of different types of buyers and monopoly cannot tell which buyer is which. The buyers differ from one another in two aspects. (1) Each buyer has a demand parameter θ, which is distributed over the set Θ with a distribution function $f(\theta)$. (2) Each buyer is either *fair* (has a utility function that depends on personal consumption as well as consumption of others) with probability γ or *neutral* (utility function only depends on personal consumption) with probability $(1-\gamma)$. Thus, the type

[9]We restrict attention to case where disutility is received only when other's net surplus is higher than own net surplus. The scope of our study is limited to this kind of other-regarding preferences. We do not focus on other types of other-regarding preferences such as the preferences that represent trust, reciprocity, altruism, and spitefulness. For instance, the preferences depicted by Bolton and Ockenfels (2000), where each agent compares own payoff to the average payoff in a reference group, and those depicted by Levine (1998) who studies altruistic and spiteful preferences are beyond the scope of this study. Also see McCabe, Rigdon, and Smith (2003) who provide an experimental analysis regarding trust and reciprocity.

space is given by $\Theta \times \{f, n\}$, where f denotes a fair type and n denotes a neutral type. The type of the buyer is private information, that is, only buyer knows her type (θ, t), where $\theta \in \Theta$ and $t \in \{f, n\}$. The utility function of a neutral buyer with θ has the form $V(q, \theta) = \theta u(q) - T(q)$, where q is the quantity consumed, $u(\cdot)$ is a strictly concave and increasing function and $T(\cdot)$ is the payment for the quantity q, made by the buyer to the monopoly. The type space and the distribution function are common knowledge.

The fair buyers have a special case of Fehr and Schmidt (1999) type utility functions. A fair type with θ receives the following net utility.

$$W(q,\theta) = V(q,\theta) - \alpha\Big[\gamma \int_{\Theta} \max\{0, V(q_f(\hat{\theta}), \hat{\theta}) - V(q,\theta)\} f(\hat{\theta}) d\hat{\theta} + (1-\gamma) \int_{\Theta} \max\{0, V(q_n(\hat{\theta}), \hat{\theta}) - V(q,\theta)\} f(\hat{\theta}) d\hat{\theta}\Big]$$

where $\alpha > 0$ is the parameter that captures fairness of the buyer, and $q_f(\hat{\theta})$ is the quantity level of a fair type with demand parameter $\hat{\theta}$, and $q_n(\hat{\theta})$ is the quantity level of a neutral type with demand parameter $\hat{\theta}$.[10] The larger the α, the larger the fairness concern of a fair type. When a fair type buyer's benefit from consumption net of payment is less than the benefit from consumption net of payment of other buyers, her net utility decreases.[11] Note that a buyer of type $\hat{\theta}$ evaluates the utility of the other types, $V(q(\theta), \theta)$, in order to calculate the disutility she gets from inequity. This is feasible since she knows the type space and the distribution function, and from the nonlinear pricing offered she can calculate which quantity-total payment bundle is targeted at which type.[12] The reason for this type of inequity which stems from the differences in net utilities may be the following. Although buyers' demand parameters are different, still the monopoly's set of offers determines the final utility levels, for instance, it is up to monopoly to extract the entire surplus

[10] We abuse notation by writing $W(q, \theta)$, as W depends on the entire menu of bundles which determine other buyers' surpluses.

[11] This version of fairness is one-sided, in the sense that an inequity-averse buyer cares about only when her payoff is less than other buyers' payoffs, and she does not care about the case where her payoff is larger than other buyers' payoff. Fehr and Schmidt (1999) provide a two sided fairness concern, where the agent cares about both types of inequities. With two sided fairness we would have

$$\begin{aligned} W(q,\theta) = V(q,\theta) - \alpha\Big[&\gamma \int_{\Theta} \max\{0, V(q_f(\hat{\theta}), \hat{\theta}) - V(q,\theta)\} f(\hat{\theta}) d\hat{\theta} \\ &+(1-\gamma) \int_{\Theta} \max\{0, V(q_n(\hat{\theta}), \hat{\theta}) - V(q,\theta)\} f(\hat{\theta}) d\hat{\theta}\Big] \\ &-\beta\Big[\gamma \int_{\Theta} \max\{0, V(q,\theta) - V(q_f(\hat{\theta}), \hat{\theta})\} f(\hat{\theta}) d\hat{\theta} \\ &+(1-\gamma) \int_{\Theta} \max\{0, V(q,\theta) - V(q_n(\hat{\theta}), \hat{\theta})\} f(\hat{\theta}) d\hat{\theta}\Big] \end{aligned}$$

where α captures fairness concern when the buyer receives less relative to others, and β captures fairness concern when the buyer receives more relative to others, where $\alpha \geq \beta$. In this paper, we assume $\alpha > \beta = 0$ and focus on fairness concern stemming from when one receives less from others. When $\beta > 0$ is allowed, we believe that the distortion we get in our setting, with $\alpha > \beta = 0$, would be even more salient. The monopoly would be forced to provide a more balanced set of utilities, relative to the case we study below. Thus, we believe that our results will not change qualitatively if we include a positive β, and it may not be worth including this type of aversion, as it makes the analysis even more cumbersome.

[12] Alternatively, we can think of type $\hat{\theta}$ calculating the utility she would get from the other types' bundles, that is, $V(q(\theta), \hat{\theta})$ and then calculate the disutility from possible inequities, looking at the difference $V(q(\hat{\theta}), \hat{\theta}) - V(q(\theta), \hat{\theta})$. We discuss this alternative way of calculating the disutility from inequity, in Section 5.3.

of the low type. After all, the monopoly does not constrain itself by making sure that all or some types end up with the same net utility. Thus, we think that it is plausible to consider buyers who can feel aversion to ending up with a lower surplus level than others, even if they know that others have different demand parameters.

The monopoly offers a set of bundles, $\{(q_i, T_i)\}_{i\in\Theta\times\{f,n\}}$. Each (q_i, T_i) is targeted at the corresponding type i. Monopoly solves the following profit maximizing problem to find the optimal nonlinear pricing, that is, optimal $\{(q_i, T_i)\}_{i\in\Theta\times\{f,n\}}$, where we use $\{q_f(\theta), T_f(\theta), q_n(\theta), T_n(\theta)\}$, instead of $\{(q_i, T_i)\}_{i\in\Theta\times\{f,n\}}$, to ease notation

$$max_{\{q_t(\theta),T_t(\theta)\}_{\theta\in\Theta,t\in\{f,n\}}} \int_\Theta [\gamma[T_f(\theta) - cq_f(\theta)] + (1-\gamma)[T_n(\theta) - cq_n(\theta)]]f(\theta)d\theta$$

subject to *individual rationality* and *incentive compatibility* constraints of each type.[13] First, we analyze the case where there is a discrete set of types in Section 3 below, with the smallest possible type space. The purpose of Section 3 is to see the implications of fairness in the simplest possible setting. Then, we consider the continuum set of types in Section 4, where we generalize the insights from Section 3.

3 Discrete Types

There are two demand types, low demand types, in λ proportion, and high demand types, in $1-\lambda$ proportion. From a bundle, (q, T), low demand types receive a net benefit of $\theta_1 u(q) - T$ and high demand types receive a net benefit of $\theta_2 u(q) - T$, where $\theta_2 > \theta_1 > 0$, and $u(\cdot)$ is a strictly increasing and strictly concave function, that is, $u' > 0$ and $u'' < 0$.

A low demand type buyer has fairness concerns with γ probability and has no fairness concerns with $1-\gamma$ probability. Thus, we have effectively three types of buyers: $\lambda\gamma$ proportion is low demand type with fairness concerns (*low type fair buyers*), $\lambda(1-\gamma)$ proportion is low demand type with no fairness concern (*neutral low type buyers*), and $(1-\lambda)$ proportion is high demand type with no fairness concern (*neutral high type buyers* or simply *high type buyers*).[14]

A low-demand buyer, who is a fair type, receives a net benefit from bundle (q, T), which is given by

$$W(q,T) = V - \alpha[\lambda\gamma \max\{V_f - V, 0\} + (1-\gamma)\lambda \max\{V_1 - V, 0\} + (1-\lambda)\max\{V_2 - V, 0\}]$$

where $V = \theta_1 u(q) - T$, $V_f = \theta_1 u(q_f) - T_f$, $V_1 = \theta_1 u(q_1) - T_1$ and $V_2 = \theta_2 u(q_2) - T_2$. Here, $\alpha > 0$ summarizes the degree of fairness concern: the larger α, the larger the disutility from inequity a buyer receives.

[13]These constraints will be given in detail in Sections 3.1 and 4.1 below for two different type spaces.

[14]Here we consider a simpler version of the main model given in Section 2 by allowing only two types of demand and only one of them being possibly a fair type. In Section 4, where we study the case of continuum types, we allow for each demand type to be possibly fair.

3.1 Optimal nonlinear pricing

The monopoly offers three bundles, (q_f, T_f), (q_1, T_1) and (q_2, T_2), targeted at low type fair buyers, neutral low type buyers and high type buyers, respectively, and then solves the following profit maximizing problem to find the optimal nonlinear pricing, $\{(q_i, T_i)\}_{i=f,1,2}$

$$max_{\{(q_i,T_i)\}_{i=f,1,2}} \lambda\gamma[T_f - cq_f] + \lambda(1-\gamma)[T_1 - cq_1] + (1-\lambda)[T_2 - cq_2]$$

subject to *individual rationality* and *incentive compatibility* constraints of each type.

Through the incentive compatibility constraints monopoly makes sure that each type chooses the bundle targeted at her. Then, a neutral high type buyer gets the bundle (q_2, T_2) and receives a net utility V_2, and a neutral low type buyer gets the bundle (q_1, T_1) and receives a net utility V_1. A fair low type buyer gets the bundle (q_f, T_f) and her net utility is

$$W(q,T) = V_f - \alpha[(1-\gamma)\lambda \max\{V_1 - V_f, 0\} + (1-\lambda)\max\{V_2 - V_f, 0\}]$$

Assuming outside options are zero[15], individual rationality for a low type fair buyer, if she gets the bundle targeted at her is given by

$$V_f - \alpha[(1-\gamma)\lambda \max\{V_1 - V_f, 0\} + (1-\lambda)\max\{V_2 - V_f, 0\}] \geq 0 \qquad (IR_f)$$

where $V_f = \theta_1 u(q_f) - T_f$ and $V_i = \theta_i u(q_i) - T_i$ for $i = 1, 2$. The individual rationality constraints for the other two (neutral) types are simply $V_1 \geq 0$ (IR_1) and $V_2 \geq 0$ (IR_2).

The incentive compatibility constraints for a low type fair buyer are as follows.

$$\begin{aligned} &V_f - \alpha[(1-\gamma)\lambda \max\{V_1 - V_f, 0\} + (1-\lambda)\max\{V_2 - V_f, 0\}] \\ &\geq V_1 - \alpha[(1-\gamma)\lambda \max\{V_1 - V_1, 0\} + (1-\lambda)\max\{V_2 - V_1, 0\}] \qquad (IC_{f,1}) \end{aligned}$$

and

$$\begin{aligned} &V_f - \alpha[(1-\gamma)\lambda \max\{V_1 - V_f, 0\} + (1-\lambda)\max\{V_2 - V_f, 0\}] \\ &\geq V_1^2 - \alpha[(1-\gamma)\lambda \max\{V_1 - V_1^2, 0\} + (1-\lambda)\max\{V_2 - V_1^2, 0\}] \qquad (IC_{f,2}) \end{aligned}$$

where $V_1^2 = \theta_1 u(q_2) - T_2$ is the net benefit low type would get if she gets the bundle targeted at the high

[15] We discuss nonzero outside option case, especially the one that would arise from the disutility from inequity when not buying any amount, in Section 5.2 for the continuum type space.

type. The incentive compatibility constraints for the neutral low type are $V_1 \geq V_1^2$ ($IC_{1,2}$) and $V_1 \geq V_f$ ($IC_{1,f}$). The incentive compatibility constraints for the high type are $V_2 \geq V_2^1$ ($IC_{2,1}$) and $V_2 \geq V_2^f$ ($IC_{2,f}$), where $V_2^1 = \theta_2 u(q_1) - T_1$ is the net benefit high type would get if she gets the bundle targeted at the neutral low type buyer, and $V_2^f = \theta_2 u(q_f) - T_f$ is the net benefit high type would get if she gets the bundle targeted at the low type fair buyer.

Assumption 1 $\theta_1 > (1 - \frac{\lambda}{1+\alpha(1-\lambda)})\theta_2$.

Assumption 1 makes sure that serving all types is optimal. Under Assumption 1, we prove the following result concerning the efficiency of the optimal nonlinear pricing the monopoly uses.

Proposition 1 *Under Assumption 1, the profit maximizing* $\{(q_i^*, T_i^*)\}_{i=f,1,2}$ *is given by*

$$\theta_1 u'(q_1^*) = \theta_1 u'(q_f^*) = c' > c. \qquad T_1^* = T_f^* = \frac{\theta_1 - K\theta_2}{1-K} u(q_1^*)$$

$$\theta_2 u'(q_2^*) = c \qquad T_2^* = \theta_2(u(q_2^*) - u(q_1^*)) + T_1^*$$

where $K = \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)} \in (0,1)$ *and* $c' = \frac{\lambda(1-K)\theta_1}{\lambda(1-K)\theta_2 - (\theta_2-\theta_1)} c$. *Moreover,* $q_2^* > q_1^* = q_f^*$ *and* $T_2^* > T_1^* = T_f^*$. *The low type fair buyers get zero surplus, while neutral low type buyers and high type buyers get positive surplus.*[16]

Proof. See Appendix 7.1. ■

In the standard model, high type receives some information rent and monopoly makes sure that high type does not deviate to low type's bundle, and the low type is left with zero surplus. When there is inequity aversion, however, there is an extra distortion in terms of participation constraint of the fair low type buyer, since she gets additional disutility when other buyers receive a higher utility from the bundle they receive. Thus, the monopoly has some incentives to provide a relatively more balanced set of utilities, and at the same time it still needs to provide incentives for each type not to deviate, especially for the high type. This tradeoff distorts the bundle for the low type and also the payment of the high type. Choosing three different bundles is too costly due to inequity aversion, thus the monopoly chooses to give the fair and neutral low types the same bundle, but a different bundle for the high type. This makes the neutral low type strictly better off relative to the case with no inequity aversion. This is because the existence of fair low type buyers forces the monopoly to leave some information rents to the neutral low type as well, in the sense that the neutral low type is not the "worst type" anymore. Now, monopoly is also forced to change the total payment of the high type to induce her not to deviate. However, there are two forces affecting her payment, an upward

[16] Note that when $\alpha = 0$ (thus, $K = 0$), we have $\theta_1 u'(q_1^*) = \frac{\lambda\theta_1}{\lambda\theta_2 - (\theta_2-\theta_1)} c$, which is equivalent to the condition in the standard case: $\theta_1 u'(q_1^*) = \frac{c}{1 - \frac{1-\lambda}{\lambda}\frac{\theta_2-\theta_1}{\theta_1}}$. See Tirole (1988), page 154 for this condition. Also, note that Assumption 1 implies $\lambda\theta_2 > \theta_2 - \theta_1$, which makes sure the denominator is positive and all types are served when there is no inequity aversion.

pressure from the lower quantity of the low type and an downward pressure from the lower payment of the low type. This tradeoff depends on the model parameters and the degree of the concavity of the utility function $u(\cdot)$. We provide a resolution to this tension in Corollary 2 below, where we show that the high type is adversely affected from inequity aversion of the fair low type, under some mild conditions.

Next, we provide a result that shows how suboptimality is affected by the degree of inequity aversion. Before we provide this result below in Proposition 2, we first explain what we mean by suboptimality in this context. In the nonlinear pricing context, a quantity level is socially optimal if its social marginal benefit is equal to its social marginal cost, otherwise it is socially suboptimal. For a neutral type i, if $\frac{\partial V}{\partial q_i} = \theta_i u'(q_i) = kc$ with $k > 0$, then q_i is socially optimal if $k = 1$ and suboptimal if $k \neq 1$. For a fair type buyer, we need to take into account her disutility from other buyers' higher surpluses as well. The net utility of a fair type buyer is summarized by W instead of V. Thus, if $\frac{\partial W}{\partial q_f} = kc$ with $k > 0$, then q_f is socially optimal if $k = 1$ and suboptimal if $k \neq 1$. Finally, we define $|k - 1|$ to be the degree of suboptimality, that is, the distance between k and the socially optimal case, which is $k = 1$.

Proposition 2 *The degree of suboptimality increases as the degree of inequity aversion, α, increases.*

Proof. We know by Proposition 1, the quantity for the high type is already socially optimal, as its social marginal benefit equals to the marginal cost: $\theta_2 u'(q_2^*) = c$.[17] Proposition 1 also shows that the suboptimality involved in the quantity for the neutral low type, q_1^*, is summarized in the difference between c and c', where $c' > c$ with $c' = kc$, that is, $k > 1$. Note that $k = \frac{\lambda(1-K)\theta_1}{\lambda(1-K)\theta_2 - (\theta_2 - \theta_1)}$, where $K = \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)} \in (0, 1)$. We need to show that k increases as α increases.

$$
\begin{aligned}
\frac{dk}{d\alpha} &= \frac{d}{dK}\left[\frac{\lambda(1-K)\theta_1 c}{\lambda(1-K)\theta_2 - (\theta_2 - \theta_1)}\right]\frac{dK}{d\alpha} \\
&= \lambda\theta_1 c \frac{d}{dK}\left[\frac{1-K}{\lambda(1-K)\theta_2 - (\theta_2 - \theta_1)}\right]\frac{(1-\lambda)(1+\alpha(1-\lambda)) - (1-\lambda)\alpha(1-\lambda)}{(1+\alpha(1-\lambda))^2} \\
&= \lambda\theta_1 c \frac{-\lambda\theta_2(1-K) + (\theta_2 - \theta_1) + \lambda\theta_2(1-K)}{(\lambda(1-K)\theta_2 - (\theta_2 - \theta_1))^2}\frac{1-\lambda}{(1+\alpha(1-\lambda))^2} \\
&= \lambda\theta_1 c \frac{\theta_2 - \theta_1}{(\lambda(1-K)\theta_2 - (\theta_2 - \theta_1))^2}\frac{1-\lambda}{(1+\alpha(1-\lambda))^2} > 0
\end{aligned}
$$

since $0 < \lambda < 1$, $\theta_2 > \theta_1 > 0$ and $c > 0$. Thus, as α increases, the suboptimality of q_1 also increases.

Now, we turn to the suboptimality involved in, q_f^*, the quantity for the fair low type buyer. Since the bundles for the fair low type and neutral low type buyers are the same, we have $V_f = V_1$. This implies that $W = V_f - \alpha(1-\lambda)(V_2 - V_f) = [1 + \alpha(1-\lambda)]V_f - \alpha(1-\lambda)V_2$. For the fair low type, the marginal benefit

[17] We already take into account the extra social cost that arises in the form of disutility fair type gets when measuring the suboptimality involved in the quantity for the fair type.

of q_f^* is then given by $\frac{\partial W}{\partial q_f} = [1+\alpha(1-\lambda)]\theta_1 u'(q_f^*)$. From Proposition 1, we know $\theta_1 u'(q_f^*) = kc$. Thus, the marginal benefit of q_f^* is given by $[1+\alpha(1-\lambda)]kc$. We just showed that k increases in α. Clearly, $1+\alpha(1-\lambda)$ also increases in α. Thus, as α increases, $[1+\alpha(1-\lambda)]k$ also increases, hence the suboptimality increases.[18] Overall, since both suboptimalities are increasing in α, the result follows. ■

The optimal quantity level for the high type buyer (q_2^*) is socially optimal, that is, marginal utility of consumption is the same as it's marginal cost. However, the quantity level for the low type fair buyer (q_f^*) and the one for the neutral low type buyer (q_1^*) are *socially suboptimal.* This is parallel to the standard case, where each type is neutral and the low type gets the quantity level which is socially suboptimal. However, in the standard case, it also happens that the type of buyer who gets a socially suboptimal quantity level is left with zero surplus, which is not the case in the current model: *The neutral low type buyer gets a socially suboptimal quantity level, yet she ends up with some positive surplus* since her individual rationality holds with strict inequality.

Proposition 2 implies that the stronger the fairness concern, the larger the social inefficiency of the optimal nonlinear pricing.[19] The reason for this result is that when there is inequity aversion, there is extra distortion on the quantity and payment bundles offered. The neutral low type receives a positive surplus, in terms of some information rent as she is not the "worst type" now. This increase in surplus for the neutral low type causes her indifference curve shift right (on a payment-quantity space), and thus it puts an extra pressure on the incentives to be provided for the high type. For the high type not to deviate the quantity for the low type must decrease, which further increases the social suboptimality.

Now, we look at the effect of the existence of inequity-averse buyers on the neutral buyers.

Corollary 1 *Under inequity aversion, neutral low type buyer receives a higher net surplus and a bundle with lower quantity and lower total payment, relative to the case with no inequity aversion.*

Proof. The net surplus of the neutral low type is zero in the case with no inequity aversion, yet Proposition 1 shows that this type of buyer ends up with a positive net surplus when there is inequity aversion. Again, from Proposition 1, we know that $\theta_1 u'(q_1^*) = \frac{\lambda(1-K)\theta_1}{\lambda(1-K)\theta_2-(\theta_2-\theta_1)}c$. In the case with no inequity aversion, that is, $\alpha = 0$, we have $\theta_1 u'(q_1) = \frac{\lambda\theta_1}{\lambda\theta_2-(\theta_2-\theta_1)}c$ (since $K = 0$). Since, $\frac{\lambda(1-K)\theta_1}{\lambda(1-K)\theta_2-(\theta_2-\theta_1)} > \frac{\lambda\theta_1}{\lambda\theta_2-(\theta_2-\theta_1)}$, we have $\theta_1 u'(q_1^*) > \theta_1 u'(q_1)$, where q_1 is the optimal quantity for the low type in the case with no inequity aversion. Since $u'(\cdot)$ is decreasing due to concavity, we immediately get $q_1^* < q_1$. We also know that $T_1^* = \frac{\theta_1-K\theta_2}{1-K}u(q_1^*)$ under inequity aversion. In the case with no inequity aversion, $T_1 = \theta_1 u(q_1)$. It is easy to see $\frac{\theta_1-K\theta_2}{1-K} < \theta_1$ using $\theta_2 > \theta_1$. Also, $u(q_1^*) < u(q_1)$ since $q_1^* < q_1$. Thus, $T_1^* < T_1$. ■

[18] Note that $[1+\alpha(1-\lambda)]k > 1$.

[19] We will show a parallel and more general result in the continuum case with no exclusion possibility, in Section 4.1.

The reason behind Corollary 1 is the following. We know that the distortion on the quantity for the low demand buyer is larger, through a smaller quantity. Also the neutral low type ends up with positive surplus when there are inequity-averse buyers, whereas she receives zero net surplus when there are no inequity-averse buyers. Thus, the total payment for this buyer should also be smaller, relative to the bundle she would get when there is no inequity-averse buyers. The comparison for the high type buyer is less straight forward, and we provide a comparison using CARA or CRRA utility functions.

Corollary 2 *Assume $\theta_2 - \theta_1 > \lambda(\theta_2 - c)$. For a CARA utility function in the form of $u(q) = 1 - e^{-q}$ or for a CRRA utility function in the form of $u(q) = log(q+1)$, under inequity aversion, high type buyer receives a lower net surplus, relative to the case with no inequity aversion.*

Proof. When there is inequity aversion, the high type buyer receives $\theta_2 u(q_2^*) - T_2^*$, and she receives $\theta_2 u(q_2) - T_2$ when there is no inequity aversion, where (q_2, T_2) is the optimal bundle for the high type in the case with no inequity aversion. Since $\theta_2 u(q_2^*) = c$ and $\theta_2 u(q_2) = c$ in both cases, it suffices to show that $T_2^* > T_2$. We know that $T_2^* = \theta_2(u(q_2^*) - u(q_1^*)) + T_1^*$ and $T_2 = \theta_2(u(q_2) - u(q_1)) + T_1$. Thus, we need $\theta_2(u(q_2^*) - u(q_1^*)) + T_1^* > \theta_2(u(q_2) - u(q_1)) + T_1$, where $T_1^* = \frac{\theta_1 - K\theta_2}{1-K} u(q_1^*)$ and $T_1 = \theta_1 u(q_1)$. Inserting these and arranging, we get that $T_2^* > T_2$ is equivalent to $(\theta_2 - \theta_1)u(q_1) > (\theta_2 - \phi)u(q_1^*)$, where $\phi = \frac{\theta_1 - K\theta_2}{1-K}$. Note that $\theta_2 - \phi = \frac{\theta_2 - \theta_1}{1-K}$. Thus, all we need to show is $(1-K)u(q_1) > u(q_1^*)$, where $K = \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)}$, and $1 - K = \frac{1}{1+\alpha(1-\lambda)}$. Thus, we need to show that $u(q_1) > (1 + \alpha(1-\lambda))u(q_1^*)$. Let $\alpha(1-\lambda) = z$, to ease notation, that is we will show $u(q_1) > (1+z)u(q_1^*)$. Now, we know that $u'(q_1^*) = \frac{\lambda(1-K)}{\lambda(1-K)\theta_2 - (\theta_2 - \theta_1)} c$ from Proposition 1. Arranging this, we have $u'(q_1^*) = \frac{\lambda}{\lambda\theta_2 - (\theta_2-\theta_1)(1+z)} c$. We also know $u'(q_1) = \frac{\lambda}{\lambda\theta_2 - (\theta_2 - \theta_1)} c$.

With CARA utility in the form of $u(q) = 1 - e^{-q}$, we have $u(q) = 1 - u'(q)$. Thus, we need to show

$$1 - \frac{\lambda c}{\lambda\theta_2 - (\theta_2 - \theta_1)} > (1+z)(1 - \frac{\lambda c}{\lambda\theta_2 - (\theta_2 - \theta_1)(1+z)})$$

Arranging this inequality, we get $\lambda^2\theta_2(\theta_2 - c) < (\theta_2 - \theta_1)[\lambda\theta_2(1+z) + \lambda\theta_2 - (\theta_2 - \theta_1)(1+z)]$. Note that $\lambda\theta_2 - (\theta_2 - \theta_1)(1+z) > 0$ by Assumption 1, and also $z = \alpha(1-\lambda) > 0$. Thus, if $\lambda^2\theta_2(\theta_2 - c) < (\theta_2 - \theta_1)\lambda\theta_2$ then we are done. This inequality is satisfied by our assumption, $\theta_2 - \theta_1 > \lambda(\theta_2 - c)$.

With CRRA utility in the form of $u(q) = log(q+1)$, we have $u'(q) = 1/(q+1)$. Thus, we need to show

$$log(\lambda\theta_2 - (\theta_2 - \theta_1)) - log(\lambda c) > (1+z)[log(\lambda\theta_2 - (\theta_2 - \theta_1)(1+z)) - log(\lambda c)]$$

which is equivalent to $zlog(\lambda c) > (1+z)log(\lambda\theta_2 - (\theta_2-\theta_1)(1+z)) - log(\lambda\theta_2 - (\theta_2-\theta_1))$. We have

$$\begin{aligned} zlog(\lambda c) &> zlog(\lambda\theta_2 - (\theta_2-\theta_1)) \\ &= (1+z)log(\lambda\theta_2 - (\theta_2-\theta_1)) - log(\lambda\theta_2 - (\theta_2-\theta_1)) \\ &> (1+z)log(\lambda\theta_2 - (\theta_2-\theta_1)(1+z)) - log(\lambda\theta_2 - (\theta_2-\theta_1)) \end{aligned}$$

The first inequality is by our assumption $(\theta_2 - \theta_1 > \lambda(\theta_2 - c))$ and the second inequality is because $\lambda\theta_2 - (\theta_2-\theta_1)(1+z) < \lambda\theta_2 - (\theta_2-\theta_1)$. ■

Corollary 2 shows that the high type buyer ends up with a higher total payment, relative to the case with no inequity aversion. This is because the quantities are distorted further for the low type buyers and monopoly leaves extra surplus to the neutral low type buyer. To compensate the monopoly extracts more from the high type (in terms of higher total payment, as the quantity for the high type does not change), which at the same time, reduces the surplus difference between these two types, hence weakening the distortion.

Proposition 3 *Monopoly profit decreases as the degree of inequity aversion, α, increases.*

Proof. By Proposition 1, we know that the optimal bundles for the fair low type and the neutral low type are the same. Thus, the profit of the monopoly is given by

$\Pi_{ineq} = \lambda\gamma[T_f^* - cq_f^*] + \lambda(1-\gamma)[T_1^* - cq_1^*] + (1-\lambda)[T_2^* - cq_2^*] = \lambda[T_1^* - cq_1^*] + (1-\lambda)[T_2^* - cq_2^*]$

Using the optimal payments from Proposition 1 and arranging Π_{ineq}, we get

$\Pi_{ineq} = \frac{\theta_1 - K(\alpha)\theta_2}{1-K(\alpha)} u(q_1^*) + (1-\lambda)\theta_2[u(q_2^*) - u(q_1^*)] - [\lambda c q_1^* + (1-\lambda)cq_2^*]$

By Envelope theorem, we have

$\frac{d\ \Pi_{ineq}}{d\alpha} = \frac{d}{dK}(\frac{\theta_1 - K\theta_2}{1-K})\ \frac{dK(\alpha)}{d\alpha} u(q_1^*)$, where $\frac{d}{dK}(\frac{\theta_1 - K\theta_2}{1-K}) = \frac{-\theta_2(1-K)+\theta_1-K\theta_2}{(1-K)^2} = \frac{\theta_1-\theta_2}{(1-K)^2} < 0$ and $\frac{dK(\alpha)}{d\alpha} > 0$.[20]

Thus, we get $\frac{d\ \Pi_{ineq}}{d\alpha} < 0$, which establishes our result. ■

The fact that monopoly's profit decreases as inequity aversion is stronger is quite intuitive. Since, now with inequity aversion, there is an extra distortion which makes providing incentives through all different bundles more costly, thus, monopoly achieves a lower profit level.

An interesting feature of the optimal nonlinear pricing is that the quantities and the payments depend on α parameter (through $K = \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)}$), but they *do not depend on γ, the fraction of the fair types.* The reason is that once the monopoly is restricted to serve all types of buyers, which is ensured by Assumption 1, the quantity levels and the payment levels turn out to be the same for the low type fair buyers and neutral

[20]We already know $\frac{dK(\alpha)}{d\alpha} > 0$ from the proof of Proposition 2.

low type buyers, as it is too costly to offer three different bundles, and it is easier to match the bundles for the low types. But then, the proportion of fair buyers does not affect the monopoly's overall profit level.

Another implication of the fact that the optimal nonlinear pricing does not depend on the fraction of the fair types, γ, is as follows. Regardless of the fraction of the fair types, in the optimal nonlinear pricing, the bundles targeted at both the neutral low type and the fair low type are the same and independent of γ. This implies that if $\gamma = 1$, that is if all low types are fair, at the optimal nonlinear pricing, the fair types would get the same bundle as they did when γ is much smaller. Thus, when a very small fraction for fair types is introduced, the optimal nonlinear pricing is as if all low types are inequity-averse.

4 Continuum of Types

In this section, we assume that the demand type θ is distributed over an interval $[\underline{\theta}, \bar{\theta}]$ according to the cumulative distribution function $F(\cdot)$ with a full support where the associated density function is $f(\cdot)$. Each buyer is either a fair type with probability γ or a neutral type with probability $1-\gamma$, regardless of her taste parameter. Thus, the type space is $\{[\underline{\theta}, \bar{\theta}]\} \times \{f, n\}$.

As in Section 2, the utility function of a neutral buyer with θ has the form $V(q,\theta) = \theta u(q) - T(q)$ and a fair type with θ receives the following net utility.

$$W(q,\theta) = V(q,\theta) - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}), \hat{\theta}) - V(q,\theta)\} f(\hat{\theta}) d\hat{\theta} + (1-\gamma) \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}), \hat{\theta}) - V(q,\theta)\} f(\hat{\theta}) d\hat{\theta}\Big]$$

4.1 Optimal Nonlinear Pricing

We consider the case where the monopoly serves to every type.[21] With no exclusion, the monopoly solves the following problem.

$$max_{\{q_t(\theta), T_t(\theta)\}_{\theta\in[\underline{\theta},\bar{\theta}], t\in\{f,n\}}} \int_{\underline{\theta}}^{\bar{\theta}} [\gamma[T_f(\theta) - cq_f(\theta)] + (1-\gamma)[T_n(\theta) - cq_n(\theta)]] f(\theta) d\theta$$

subject to *individual rationality* and *incentive compatibility* constraints of each type. We begin with describing the individual rationality and the incentive compatibility constraints.

Individual Rationality: Each type of buyer should be at least as well off by choosing the bundle targeted at her as her outside option. Assuming outside options are zero, individual rationality constraints

[21] We also provide an analysis, in Section 5.1, for the case where the monopoly is allowed to choose whom to serve and whom not to, that is, where exclusion is possible. Solving that extension is technically cumbersome, however, we show that even if the monopoly is allowed to exclude some types, it is optimal not to exclude any types, thus the optimal nonlinear pricing turns out to be the one we provide here in this section.

for neutral and fair types are:

$$IR^n(\theta): \quad \theta u(q_n(\theta)) - T(q(\theta)) \geq 0$$

$$IR^f(\theta): \quad V(q_f(\theta),\theta) - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta}) d\hat{\theta}$$

$$+(1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta}) d\hat{\theta}\Big] \geq 0$$

Incentive Compatibility: Each type of buyer should be at least as well off by choosing the bundle targeted at her as choosing another bundle. A neutral buyer should not mimic other neutral types, $IC^{n,n}$, as well as other fair types, $IC^{n,f}$.

$$IC^{n,n}(\theta): \quad \theta u(q_n(\theta)) - T(q_n(\theta)) \geq \theta u(q_n(\theta')) - T(q_n(\theta'))$$

and

$$IC^{n,f}(\theta): \quad \theta u(q_n(\theta)) - T(q_n(\theta)) \geq \theta u(q_f(\theta')) - T(q_f(\theta'))$$

for all $\theta \in [\underline{\theta}, \bar{\theta}]$. A fair buyer should not mimic other fair types, $IC^{f,f}$, as well as other neutral types, $IC^{f,n}$.

$$IC^{f,n}(\theta): \quad V(q_f(\theta),\theta) - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta}) d\hat{\theta}$$

$$+(1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta}) d\hat{\theta}\Big]$$

$$\geq V(q_n(\theta'),\theta) - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta}) - V(q_n(\theta'),\theta)\} f(\hat{\theta}) d\hat{\theta}$$

$$+(1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta}) - V(q_n(\theta'),\theta)\} f(\hat{\theta}) d\hat{\theta}\Big]$$

and

$$IC^{f,f}(\theta): \quad V(q_f(\theta),\theta) - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta}) d\hat{\theta}$$

$$+(1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta}) d\hat{\theta}\Big]$$

$$\geq V(q_f(\theta'),\theta) - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta'),\theta)\} f(\hat{\theta}) d\hat{\theta}$$

$$+(1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta}) - V(q_f(\theta'),\theta)\} f(\hat{\theta}) d\hat{\theta}\Big]$$

for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Thus, we have four different sets of incentive compatibility constraints.

Assumption 2 *$V(q,\theta)$ is strictly concave in q.*[22]

Lemma 1 *Under Assumption 2, the monopoly's problem reduces to*

$$max_{\{q_f(\theta),T(q_f(\theta))\}_{\theta\in[\underline{\theta},\bar{\theta}]}}\int_{\underline{\theta}}^{\bar{\theta}}[T(q_f(\theta))-cq_f(\theta)]f(\theta)d\theta$$

subject to $IR^f(\underline{\theta}):V(q_f(\underline{\theta}),\underline{\theta})-\alpha[\int_{\underline{\theta}}^{\bar{\theta}}(V(q_f(\hat{\theta}),\hat{\theta})-V(q_f(\underline{\theta}),\underline{\theta}))f(\hat{\theta})d\hat{\theta}]=0$, *and*

$IC^{n,n}(\theta):\theta u'(q_f(\theta))-T'(q_f(\theta))=0$ *for all* θ.

Proof. This is proven through a series of lemmas in Appendix 7.2. ■

Before we characterize the quantities in the optimal nonlinear pricing when exclusion is not possible, we make two assumptions.

Assumption 3 *Hazard rate of F increases with θ, that is, $\frac{f(\theta)}{1-F(\theta)}$ is increasing in θ.*

Assumption 3 is fairly reasonable and common. It is satisfied by a number of distributions, including the normal, the uniform, the logistic and the exponential and any distribution with nondecreasing density function.

Assumption 4 $\frac{\underline{\theta}f(\underline{\theta})}{1-F(\underline{\theta})}>1+\alpha$.

Assumption 4 is the counterpart of Assumption 1 in the discrete case. Note that Assumption 3 and 4 together imply $\frac{\theta f(\theta)}{1-F(\theta)}>1+\alpha$ for all θ, which in turn implies $\theta-(1+\alpha)H(\theta)>0$ for any θ, where $H(\theta)=\frac{1-F(\theta)}{f(\theta)}$.

Proposition 4 *In the optimal nonlinear pricing when exclusion is not allowed, the quantities are $q_n^*(\theta)=q_f^*(\theta)=q^*(\theta)$, where $q^*(\theta)$ solves $u'(q(\theta))[\theta-H(\theta)-\alpha H(\theta)]=c$, for every θ.*

Proof. First, we have $q_n^*(\theta)=q_f^*(\theta)=q^*(\theta)$ from Lemma 23 in Appendix 7.2. From the envelope theorem and $IC^{n,n}(\theta)$, we have $\frac{\partial V}{\partial\theta}=V'(\theta)=u(q(\theta))$. By integrating it, we get:

$$V(q(\theta),\theta)=\int_{\underline{\theta}}^{\theta}u(q(\hat{\theta}))d\hat{\theta}+V(q(\underline{\theta}),\underline{\theta})$$

We also have $W(\underline{\theta})=0$ since $IR^f(\underline{\theta})$ is binding. Then, we have

$$V(q(\underline{\theta}),\underline{\theta})=\alpha[\int_{\underline{\theta}}^{\bar{\theta}}(V(q(\hat{\theta}),\hat{\theta})-V(q(\underline{\theta}),\underline{\theta})f(\hat{\theta})d\hat{\theta}]=\alpha[\int_{\underline{\theta}}^{\bar{\theta}}\Big(\int_{\underline{\theta}}^{\hat{\theta}}u(q(t))dt\Big)f(\hat{\theta})d\hat{\theta}]=\alpha[\int_{\underline{\theta}}^{\bar{\theta}}u(q(\theta))(1-F(\theta))d\theta]$$

[22] We verify that this assumption actually holds under the optimal payment scheme $T(q)$, right after the proof of Corollary 3 below.

Since $q_n(\theta) = q_f(\theta)$, we have $T(q_f(\theta)) = T(q_n(\theta))$. Using $T(q(\theta)) = \theta u(q(\theta)) - V(\theta)$, the profit of the monopoly becomes

$$\Pi = \int_{\underline{\theta}}^{\bar{\theta}} \left(\theta u(q(\theta)) - \int_{\underline{\theta}}^{\theta} u(q(\hat{\theta}))d\hat{\theta} - \alpha[\int_{\underline{\theta}}^{\bar{\theta}} u(q(\theta))(1-F(\theta))d\theta] - cq(\theta) \right) f(\theta)d\theta$$

Integrating by parts, we get

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(\int_{\underline{\theta}}^{\theta} u(q(\hat{\theta}))d(\hat{\theta}) \right) f(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} u(q(\theta))(1-F(\theta))d\theta$$

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(\int_{\underline{\theta}}^{\bar{\theta}} u(q(\theta))(1-F(\theta))d\theta \right) f(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} u(q(\theta))(1-F(\theta))d\theta$$

Hence, the profit becomes

$$\Pi = \int_{\underline{\theta}}^{\bar{\theta}} \left(\theta u(q(\theta)) - u(q(\theta))\frac{(1-F(\theta))}{f(\theta)} - \alpha u(q(\theta))\frac{(1-F(\theta))}{f(\theta)} - cq(\theta) \right) f(\theta)d\theta$$

Inserting $H(\theta) = \frac{1-F(\theta)}{f(\theta)}$, we maximize the integral pointwise. The first order condition with respect to q at a fixed θ gives $\theta u'(q(\theta)) - u'(q(\theta))H(\theta) - \alpha u'(q(\theta))H(\theta) - c = 0$. Rearranging, we get

$$u'(q(\theta))[\theta - (1+\alpha)H(\theta)] = c \tag{1}$$

which characterizes the quantity function in the optimal nonlinear pricing.[23] ■

The above result says that there is pooling, that is, neutral and fair types of the same demand parameter receives the same quantity. The intuition is similar to the one in the discrete case. Because of the inequity aversion of the fair type, it is costlier to give right incentives for each buyer to self select, by offering different bundles. A direct corollary to Proposition 4 is that the larger the fairness concern, the larger the suboptimality involved in the optimal nonlinear pricing, which implies that the quantity offered decreases.

Corollary 3 *As α increases, the degree of suboptimality increases and the quantity offered $q^*(\theta)$ decreases for any θ.*

Proof. Consider the suboptimality involved in the quantities for the neutral types. By Equation 1 we have $\theta u'(q^*(\theta)) = c\ \frac{\theta}{\theta-(1+\alpha)H(\theta)} > c$. For any θ, the fraction $\frac{\theta}{\theta-(1+\alpha)H(\theta)}$ is positive by Assumption 3 and 4 and increasing as α increases. Thus, the right hand side also increases, which means a higher suboptimality.

[23] Note that when $\alpha = 0$, Equation 1 is equivalent to the one in the standard model. See equation (3.14) on page 156, in Tirole (1988). Also, note that Assumption 3 and 4 makes sure that $\theta - (1+\alpha)H(\theta) > 0$, which is needed since $u'(q) > 0$.

Now consider the suboptimality involved in the quantities for the fair types. Since, $q_n(\theta) = q_f(\theta) = q(\theta)$ by Proposition 4, and since $V(q(\theta), \theta)$ is increasing in θ by Lemma 14, we have

$$
\begin{aligned}
W &= V(q,\theta) - \alpha\Big[\gamma \int_{\theta}^{\bar{\theta}} [V(q(\hat{\theta}),\hat{\theta}) - V(q,\theta)] f(\hat{\theta}) d\hat{\theta} + (1-\gamma) \int_{\theta}^{\bar{\theta}} [V(q(\hat{\theta}),\hat{\theta}) - V(q,\theta)] f(\hat{\theta}) d\hat{\theta}\Big] \\
&= V(q,\theta) - \alpha\Big[\gamma \int_{\theta}^{\bar{\theta}} V(q(\hat{\theta}),\hat{\theta}) f(\hat{\theta}) d\hat{\theta} - \gamma[\bar{\theta} - \theta] V(q,\theta) + (1-\gamma) \int_{\theta}^{\bar{\theta}} V(q(\hat{\theta}),\hat{\theta}) f(\hat{\theta}) d\hat{\theta} - (1-\gamma)[\bar{\theta} - \theta] V(q,\theta)\Big] \\
&= (1 + \alpha[\bar{\theta} - \theta]) V(q,\theta) - \alpha \int_{\theta}^{\bar{\theta}} V(q(\hat{\theta}),\hat{\theta}) f(\hat{\theta}) d\hat{\theta}
\end{aligned}
$$

The marginal benefit for a fair type buyer with θ is then given by

$$
\frac{\partial W}{\partial q_f} = (1 + \alpha[\bar{\theta} - \theta]) \frac{\partial V}{\partial q_f} = (1 + \alpha[\bar{\theta} - \theta]) \theta u'(q^*(\theta)) = (1 + \alpha[\bar{\theta} - \theta]) \frac{\theta}{\theta - (1+\alpha) H(\theta)} c = kc
$$

Since both $(1 + \alpha[\bar{\theta} - \theta])$ and $\frac{\theta}{\theta - (1+\alpha)H(\theta)}$ are increasing in α, we conclude that the suboptimality, $|k-1|$, increases in α. Finally, since $\theta u'(q^*(\theta))$ is larger when α is larger, we immediately get a smaller $q^*(\theta)$ for any θ since u is strictly concave. ■

Note that the above result is also valid at $\alpha = 0$. Thus, the suboptimality is larger when there is inequity aversion relative to the case where there is no inequity aversion.

Verification of Assumption 2: We verify that $V(q,\theta) = \theta u(q) - T(q)$ is strictly concave in q under the optimal payment scheme $T(q)$. To see this, take two quantity functions, $\overline{q}(\cdot)$ and $\overline{\overline{q}}(\cdot)$ and a real number $\delta \in (0,1)$. For any given θ, from the proof of Proposition 4, we have

$$
\begin{aligned}
& V(\delta\overline{q}(\theta) + (1-\delta)\overline{\overline{q}}(\theta), \theta) = \int_{\underline{\theta}}^{\theta} u(\delta\overline{q}(\hat{\theta}) + (1-\delta)\overline{\overline{q}}(\hat{\theta})) d\hat{\theta} + \alpha \int_{\underline{\theta}}^{\bar{\theta}} u(\delta\overline{q}(\hat{\theta}) + (1-\delta)\overline{\overline{q}}(\hat{\theta}))(1 - F(\hat{\theta})) d\hat{\theta} \\
> & \int_{\underline{\theta}}^{\theta} [\delta u(\overline{q}(\hat{\theta})) + (1-\delta) u(\overline{\overline{q}}(\hat{\theta}))] d\hat{\theta} + \alpha \int_{\underline{\theta}}^{\bar{\theta}} [\delta u(\overline{q}(\hat{\theta})) + (1-\delta) u(\overline{\overline{q}}(\hat{\theta}))](1 - F(\hat{\theta})) d\hat{\theta} \\
= & \delta\Big[\int_{\underline{\theta}}^{\theta} u(\overline{q}(\hat{\theta})) d\hat{\theta} + \alpha \int_{\underline{\theta}}^{\bar{\theta}} u(\overline{q}(\hat{\theta}))(1 - F(\hat{\theta})) d\hat{\theta}\Big] + (1-\delta)\Big[\int_{\underline{\theta}}^{\theta} u(\overline{\overline{q}}(\hat{\theta})) d\hat{\theta} + \alpha \int_{\underline{\theta}}^{\bar{\theta}} u(\overline{\overline{q}}(\hat{\theta}))(1 - F(\hat{\theta})) d\hat{\theta}\Big] \\
= & \delta V(\overline{q}(\theta), \theta) + (1-\delta) V(\overline{\overline{q}}(\theta), \theta)
\end{aligned}
$$

which establishes that $V(q,\theta)$ is strictly concave in q.

Proposition 5 *There exists a $\tilde{\theta} \in (\underline{\theta}, \bar{\theta}]$, such that for all $\theta < \tilde{\theta}$, neutral types are strictly better off when there are fair types relative to when there are none.*

Proof. From the proof of Proposition 4, we have $V(q^*(\underline{\theta}), \underline{\theta}) = \alpha[\int_{\underline{\theta}}^{\bar{\theta}} u(q^*(\theta))(1 - F(\theta)) d\theta] > 0$.[24] Note that

[24]This can also be seen by $IR^f(\underline{\theta})$ and Lemma 22, where we have $W(q^*(\underline{\theta}), \underline{\theta}) = V(q^*(\underline{\theta}), \underline{\theta}) - \alpha \int_{\underline{\theta}}^{\bar{\theta}} [V(q^*(\hat{\theta}), \hat{\theta}) -$

when $\alpha = 0$, $V(q(\underline{\theta}), \underline{\theta}) = 0$ by $IR^n(\underline{\theta})$, where $q(\cdot)$ is the optimal quantity function in this standard case. Thus, we have $V(q^*(\underline{\theta}), \underline{\theta}) > V(q(\underline{\theta}), \underline{\theta})$. By Lemma 14, $V(q^*, \theta)$ is strictly increasing in θ. In the standard case, $V(q, \theta)$ also strictly increases in θ. Regardless of which one increases faster, since both are strictly increasing and since $V(q^*(\underline{\theta}), \underline{\theta}) > 0 = V(q(\underline{\theta}), \underline{\theta})$, there always exists a $\tilde{\theta} \in (\underline{\theta}, \bar{\theta}]$, such that for all $\theta < \tilde{\theta}$, $V(q^*, \theta) > V(q, \theta)$.[25] ■

The above proposition is the counterpart of the result in the discrete case that shows that the neutral low demand type buyer is better off. The intuition is that the existence of fair type buyers forces the monopoly to leave more positive information rent to those types that have low enough demand parameters, through pooling of bundles in order to decrease the disutility the fair types get.

Proposition 6 *Monopoly profit decreases as the degree of inequity aversion, α, increases.*

Proof. The final form of profit is

$$\Pi = \int_{\underline{\theta}}^{\bar{\theta}} \Bigg(u(q(\theta))[\theta - (1+\alpha)H(\theta)] - cq(\theta) \Bigg) f(\theta) d\theta$$

We show that $u(q(\theta))[\theta - (1+\alpha)H(\theta)] - cq(\theta)$ is decreasing in α, for any given θ. Note that for a given θ, $q(\theta)$ is decreasing in α. By Equation 1, we have $c = u'(q(\theta))[\theta - (1+\alpha)H(\theta)]$, which implies

$$\begin{aligned} u(q(\theta))[\theta - (1+\alpha)H(\theta)] - cq(\theta) &= u(q(\theta))[\theta - (1+\alpha)H(\theta)] - q(\theta)u'(q(\theta))[\theta - (1+\alpha)H(\theta)] \\ &= [\theta - (1+\alpha)H(\theta)][u(q(\theta)) - q(\theta)u'(q(\theta))] \end{aligned}$$

The expression in the first brackets decreases in α. Let's check the expression in the second brackets. $\frac{\partial[u(q) - qu'(q)]}{\partial q} = u'(q) - u'(q) - qu''(q) = -qu''(q) \geq 0$ since $u'' < 0$. Thus, this expression is increasing in q. We know that as α increases $q(\theta)$ decreases. Thus, this expression is also decreasing in α. Thus, $u(q(\theta))[\theta - (1+\alpha)H(\theta)] - cq(\theta)$ is decreasing in α, proving our result. Alternatively, note that the maximized profit of the monopoly (from the proof of Proposition 4) is given by

$$\Pi_{ineq} = \int_{\underline{\theta}}^{\bar{\theta}} \Bigg(\theta u(q^*(\theta)) - u(q^*(\theta))H(\theta) - \alpha u(q^*(\theta))H(\theta) - cq^*(\theta) \Bigg) f(\theta) d\theta$$

By Envelope theorem, the overall effect of α on the profit is $\frac{d\Pi}{d\alpha}$, which is negative, proving our result. ■

$V(q^*(\underline{\theta}), \underline{\theta})]f(\hat{\theta})d\hat{\theta} = 0$. Note that $\alpha \int_{\underline{\theta}}^{\bar{\theta}} [V(q^*(\hat{\theta}), \hat{\theta}) - V(q^*(\underline{\theta}), \underline{\theta})]f(\hat{\theta})d\hat{\theta} > 0$. This is implied by Lemma 14. Thus, we have $V(q^*(\underline{\theta}), \underline{\theta}) > 0$.

[25] For instance, if $V(q^*, \theta)$ increases faster than $V(q, \theta)$, then $\tilde{\theta} = \bar{\theta}$, and every neutral type is strictly better off when there are fair types in the environment, relative to the one with no fair types. Otherwise, $\tilde{\theta}$ may be an intermediate threshold, that is, $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$, and for any neutral type with $\theta < \tilde{\theta}$, it is better to have fair types in the environment.

The above result generalizes Proposition 3 to the continuum type case through a similar intuition. Also, as we discussed at the end of Section 3, γ does not affect the optimal nonlinear pricing, when exclusion is not allowed. Note that γ is not present in Equation 1. This is because, when the monopoly has to serve all types, then the quantities turn out to be the same for a fair and neutral types with the same taste parameter θ. Then, the probability γ disappears from the monopoly's objective function.

5 Discussion and Extensions

In this section, we discuss a number of extensions and some issues, including the optimal nonlinear pricing when the monopoly is allowed to exclude consumers, an alternative participation constraint, the way disutility from inequity is evaluated and an interpretation regarding information structure.

5.1 Optimal nonlinear pricing with exclusion

Suppose the monopoly is allowed to exclude some of the types. Assume that the monopoly sells to the neutral types with taste parameter $\theta \in [\tilde{\theta}_n, \overline{\theta}]$ and to the fair types with taste parameter $\theta \in [\tilde{\theta}_f, \overline{\theta}]$, where $\tilde{\theta}_n$ and $\tilde{\theta}_f$ are chosen by the monopoly.

We show that exclusion is not optimal under individual rationality and incentive compatibility constraints. Therefore, pricing in Section 4.1 is the optimal nonlinear pricing for the monopoly, even if exclusion is allowed. We first show that if the monopoly excludes some of the neutral types and some of the fair types, it must exclude the same set of demand types of fair and neutral types, that is, $\tilde{\theta}_f = \tilde{\theta}_n$. The idea is that if, for a given demand type θ, exactly one of the fair and neutral types is excluded and the other is included, then the excluded type will mimic the other included type. Then, we show that one of the excluded neutral types would mimic the smallest demand type who is not excluded, to get a positive payoff, as the lowest fair type gets a positive benefit $V(q, \tilde{\theta}_f)$ (that compensates the disutility from inequity to yield $W(q, \tilde{\theta}_f) = 0$). Thus, there is no incentive feasible nonlinear pricing when a positive mass of types are excluded. This leads to our conclusion of no exclusion being optimal. The analysis and the proofs are in Appendix 7.3.

5.2 An alternative individual rationality constraint

In our model, for a fair type buyer, we have considered an individual rationality constraint in which the outside option is normalized to zero. Thus, when a fair type buyer does not accept any of the offers, then she ends up with zero net payoff, and there is no additional disutility from other buyers having some positive consumption level. However, when a fair type buyer decides not to accept any of the offers, she may do so by taking into account that when she consumes none, she will end up with the disutility she may get due to her fairness concern, given that at least some other buyer ends up with some positive consumption level. Thus, an alternative IR constraint for fair type buyers may be formalized as follows.

$$IR^f(\theta): \quad V(q_f(\theta),\theta) - \alpha\Big[\gamma\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta})d\hat{\theta}$$

$$+(1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta})d\hat{\theta}\Big] \geq$$

$$-\alpha\Big[\gamma\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta})\} f(\hat{\theta})d\hat{\theta} + (1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta})\} f(\hat{\theta})d\hat{\theta}\Big]$$

where the right hand side of the inequality reflects the fair type buyer's net payoff which consists of only the disutility she gets from the case in which she does not buy at all.

Lemma 22 in Appendix 7.2 implies that $V(q_n(\theta),\theta) = V(q_f(\theta),\theta)$ for all θ, which does not use any individual rationality constraint. Thus, it is independent of this alternative version of individual rationality. Using this together with Lemma 14, we have $V(q_n(\hat{\theta}),\hat{\theta}) = V(q_f(\hat{\theta}),\hat{\theta}) > V(q_f(\theta),\theta)$ for all $\hat{\theta} > \theta$. Then, arranging the inequality above, we get

$$IR^f(\theta): \quad V(q_f(\theta),\theta) - \alpha\Big[\gamma\int_{\theta}^{\bar{\theta}} [V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)] f(\hat{\theta})d\hat{\theta} + (1-\gamma)\int_{\theta}^{\bar{\theta}} [V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)] f(\hat{\theta})d\hat{\theta}\Big]$$

$$\geq -\alpha\Big[\gamma\int_{\underline{\theta}}^{\bar{\theta}} V(q_f(\hat{\theta}),\hat{\theta}) f(\hat{\theta})d\hat{\theta} + (1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} V(q_f(\hat{\theta}),\hat{\theta}) f(\hat{\theta})d\hat{\theta}\Big]$$

which implies

$$IR^f(\theta): \quad [1+\alpha(\bar{\theta}-\theta)]V(q_f(\theta),\theta) - \alpha\int_{\theta}^{\bar{\theta}} V(q_f(\hat{\theta}),\hat{\theta}) f(\hat{\theta})d\hat{\theta} + \alpha\int_{\underline{\theta}}^{\bar{\theta}} V(q_f(\hat{\theta}),\hat{\theta}) f(\hat{\theta})d\hat{\theta} \geq 0 \tag{2}$$

The IR constraint we employed in our model for a fair type with θ with zero outside option was as follows.

$$V(q_f(\theta),\theta) - \alpha\Big[\gamma\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta})d\hat{\theta}$$

$$+(1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)\} f(\hat{\theta})d\hat{\theta}\Big] \geq 0$$

Again using $V(q_n(\hat{\theta}),\hat{\theta}) = V(q_f(\hat{\theta}),\hat{\theta}) > V(q_f(\theta),\theta)$ for all $\hat{\theta} > \theta$, and arranging we get,

$$V(q_f(\theta),\theta) - \alpha\Big[\gamma\int_{\theta}^{\bar{\theta}} [V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)] f(\hat{\theta})d\hat{\theta} + (1-\gamma)\int_{\theta}^{\bar{\theta}} [V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\theta),\theta)] f(\hat{\theta})d\hat{\theta}\Big] \geq 0$$

which can be rewritten as

$$[1+\alpha(\overline{\theta}-\theta)]V(q_f(\theta),\theta)-\alpha\int_{\theta}^{\bar{\theta}}[V(q_f(\hat{\theta}),\hat{\theta})]f(\hat{\theta})d\hat{\theta}\geq 0 \tag{3}$$

Thus, the alternative IR constraint for θ-type (inequality 2) has an extra term on its left hand side, relative to the IR constraint we used for the same type (inequality 3): $\alpha\int_{\underline{\theta}}^{\bar{\theta}}V(q_f(\hat{\theta}),\hat{\theta})f(\hat{\theta})d\hat{\theta}$. The existence of this extra term makes sense, since in the alternative version there is more room for disutility from inequity, thus the net benefit for the buyer to participate must be higher. Hence, there is this extra term on the left hand side. Also note that, this term emerges from the disutility from other buyers' benefit levels relative to not consuming at all, thus it depends only on other types' benefits, and it does not depend on θ.

However, with this alternative formulation our results do not qualitatively change. The intuition is that this extra term (independent of type θ) increases the level of inequity aversion for every fair type by the same amount, while not affecting the incentive compatibilities except, $IC^{n,f}(\theta)$, the one that makes sure the neutral type does not mimic the fair type. Since this extra term produces an extra distortion through the individual rationality condition and forces the monopoly to offer a better bundle for the fair type, it makes it harder to provide incentives for the neutral type. There will be further inefficiency, which creates larger suboptimality. Note that this term increases as α increases, thus, our result that says the degree of suboptimality increases as the degree of inequity aversion increases is still valid, now with a stronger effect. Also, the existence of this extra term translates into even smaller profit for the monopoly as well, since the information rent left to the higher types will be larger. When α is larger, this negative effect of this extra term on monopoly's profit is also larger. Finally, regarding the neutral types who are better off with the existence of fair types, these neutral types will be better off even more when this alternative individual constraint is considered, since now a better bundle will be targeted at fair types, and since there is pooling across the fair and neutral types of the same demand type, the corresponding neutral type will benefit from the existence of fair types even more.

5.3 Evaluating the disutility from inequity

When calculating a $\hat{\theta}$ type's disutility from possible inequities, we assumed that this type can calculate the utility of every other type, $V(q(\theta),\theta)$. This is feasible since she can technically learn which bundle is for which type by looking at the nonlinear pricing and she also knows the type space and the distribution function. Then, she compares $V(q(\theta),\theta)$ with her utility $V(q(\hat{\theta}),\hat{\theta})$. However, an alternative way of thinking about how to calculate the disutility from possible inequities, is to let type $\hat{\theta}$ buyer calculate the utility she would get from the bundle of other types, $V(q(\theta), \hat{\theta})$, and then look at the difference $V(q(\hat{\theta}), \hat{\theta}) - V(q(\theta), \hat{\theta})$. With this type of evaluation, the problem is that whenever $V(q(\hat{\theta}), \hat{\theta}) - V(q(\theta), \hat{\theta}) < 0$, that is, when a fair type with $\hat{\theta}$ suffers from inequity, she would deviate to get the θ type's bundle, which not only gives her a higher utility, but also decreases her disutility from possible inequities. Thus, there would be no inequity at all, and the optimal nonlinear pricing would be the same as in the standard model with no fairness issues. Instead, we took an approach where the disutility from inequity emerges from different net surpluses each type ends up with, rather than the difference in bundles picked by each type.

5.4 Inequity aversion or lack of information

Another interpretation of the distinction between fair and neutral type consumers can be their information status. Instead of assuming people with different fairness concerns, we can think that they are all inequity-averse but they only learn other people's consumption bundle with some probability, γ. For example, one would not be aware of the details of price discrimination in her flight if she does not check ticket prices at different times and locations or talk with other passengers. Therefore, even though one has fairness concerns, she may not be adversely affected by the inequity involved in price discrimination. This alternative view on parameter γ allows us to explain a different setup with the same model.

6 Conclusion

We considered both a discrete and continuum type environments where a monopoly is using a nonlinear pricing scheme facing buyers who may differ in their valuations and in their inequity preferences, in the sense that some are inequity-averse (fair types) some are not (neutral types). We adopted the utility function that takes inequity aversion into account provided by Fehr and Schmidt (1999). We characterized the optimal nonlinear pricing assuming all types are served and also provided an analysis for the case where the monopoly is allowed to exclude some buyers. The suboptimality already present in the standard nonlinear pricing problem, is larger when there are fair types in the buyer population. Monopoly is worse off, but some or all of the neutral types are better off when fair types are introduced.

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7 Appendix

7.1 Appendix A

Proof of Proposition 1. We prove Proposition 1 through a series of lemmas, Lemma 2 through 13. We start with assuming $V_2 > V_1$ and $V_2 > V_f$, which will be verified in the end of the proof.

Lemma 2 *$V_1 = V_f$ and both $IC_{1,f}$ and $IC_{f,1}$ bind.*

Proof. If $V_f \leq V_1$, then $IC_{f,1}$ becomes

$$V_f - \alpha[(1-\gamma)\lambda(V_1 - V_f) + (1-\lambda)(V_2 - V_f)] \geq V_1 - \alpha[(1-\lambda)(V_2 - V_1)]$$

which is $(V_1 - V_f)(1 + \alpha[(1-\gamma)\lambda + (1-\lambda)]) \leq 0$, implying $V_1 \leq V_f$. Thus, if $V_f \leq V_1$, we need to have $V_1 = V_f$. Now, if $V_f \geq V_1$, then $IC_{f,1}$ becomes

$$V_f - \alpha[(1-\lambda)(V_2 - V_f)] \geq V_1 - \alpha[(1-\lambda)(V_2 - V_1)]$$

which is $(V_f - V_1)(1 + \alpha(1-\lambda)) \geq 0$, which is consistent with $V_f \geq V_1$. Thus, $V_f < V_1$ is ruled out. Now, $V_f \geq V_1$ together with $IC_{1,f}$, which is $V_1 \geq V_f$, we get $V_1 = V_f$. Therefore, both $IC_{1,f}$ and $IC_{f,1}$ bind. ■

Lemma 3 *$IC_{f,2}$ is redundant.*

Proof. Note that, $V_2 - V_1^2 = \theta_2 u(q_2) - T_2 - [\theta_1 u(q_2) - T_2] = (\theta_2 - \theta_1)u(q_2) \geq 0$ since $\theta_2 > \theta_1$. Also, by $IC_{1,2}$ we have $V_1 \geq V_1^2$. Then, using these together with Lemma 2, $(IC_{f,2})$ becomes

$$V_f - \alpha(1-\lambda)(V_2 - V_f) \geq V_1^2 - \alpha[(1-\gamma)\lambda(V_1 - V_1^2) + (1-\lambda)(V_2 - V_1^2)]$$

Simplfying this, we get $(V_f - V_1^2)(1 + \alpha(1-\lambda)) \geq (V_1^2 - V_1)(\alpha\lambda(1-\gamma))$. This holds since $V_1^2 - V_1 \leq 0$ and $V_f - V_1^2 \geq 0$. The first inequality is $IC_{1,2}$ and the second one follows from $V_f = V_1 \geq V_1^2$. ■

Lemma 4 *Both IR_1 and IR_2 hold with strict inequality.*

Proof. If IR_1 binds, that is, if $V_1 = 0$ then by Lemma 1, $V_f = 0$. Then, IR_f would be $V_f - \alpha(1-\lambda)(V_2 - V_f) < 0$, since $V_2 > V_f$. Thus, IR_1 cannot bind. Now assume, IR_2 binds. Then, $V_2 = 0$, which implies $V_1 < 0$ since $V_2 > V_1$. Thus, IR_2 does not bind. ■

Note that IR_f boils down to $V_f - \alpha(1-\lambda)(V_2 - V_f) \geq 0$, which is equivalent to $V_f \geq KV_2$ where $K = \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)} \in (0,1)$. Thus, the problem is

$$max_{\{(q_i,T_i)\}_{i=f,1,2}} \lambda\gamma[T_f - cq_f] + \lambda(1-\gamma)[T_1 - cq_1] + (1-\lambda)[T_2 - cq_2]$$

subject to $V_1 = V_f \geq KV_2$, $V_1 \geq V_1^2$, $V_2 \geq V_2^1$ and $V_2 \geq V_2^f$.

We will ignore $V_1 \geq V_1^2$ for the moment and will characterize the optimal non-liner pricing in the reduced problem without this constraint. Then, after characterizing the optimal nonlinear pricing, I will show that $V_1 > V_1^2$ actually holds.

Lemma 5 *$V_1 = V_f \geq KV_2$ holds with equality.*

Proof. Otherwise, $\theta_1 u(q_1) - T_1 > K\theta_2 u(q_2) - KT_2$. Then, monopoly can increase all of the T_1, T_f and T_2 by a small enough $\epsilon > 0$, still satisfy $V_1 = V_f \geq KV_2$, increase profits and not violate any of the other constraints. Note that $\theta_1 u(q_1) - T_1 - \epsilon \geq K\theta_2 u(q_2) - KT_2 - K\epsilon$, that is, $\theta_1 u(q_1) - T_1 \geq K\theta_2 u(q_2) - KT_2 + \epsilon(1-K)$ for small enough $\epsilon > 0$, where $0 < K < 1$. Also note that $V_2 \geq V_2^1$ and $V_2 \geq V_2^f$ are not violated since both sides of these inequalities are increased by the same amount ϵ. Also, $V_1 > V_1^2$ is not violated for small enough ϵ because of the strict inequality. ■

Lemma 6 *If $q_1 > q_f$ then $V_2 = V_2^1 > V_2^f$.*

Proof. $V_1 = V_f$ implies $\theta_1 u(q_1) - T_1 = \theta_1 u(q_f) - T_f$. Thus, $T_1 - T_f = \theta_1(u(q_1) - u(q_f))$. Thus, $q_1 > q_f$ if and only if $T_1 > T_f$. Now, $V_2^1 - V_2^f = \theta_2 u(q_1) - T_1 - \theta_2 u(q_f) + T_f = \theta_2(u(q_1) - u(q_f)) - (T_1 - T_f) = \theta_2(u(q_1) - u(q_f)) - \theta_1(u(q_1) - u(q_f)) = (\theta_2 - \theta_1)(u(q_1) - u(q_f))$. Thus, $q_1 > q_f$ if and only if $V_2^1 > V_2^f$. Thus, if $q_1 > q_f$, we have $V_2 \geq V_2^1 > V_2^f$. Now, suppose $V_2 > V_2^1$, which is $\theta_2 u(q_2) - T_2 > \theta_2 u(q_1) - T_1$. Then, increasing T_2 by ϵ/K and increasing T_1 by ϵ, for small enough ϵ, we get $\theta_2 u(q_2) - T_2 - (\epsilon/K) \geq \theta_2 u(q_1) - T_1 - \epsilon$. Doing this satisfies $V_1 = V_f \geq KV_2$. It also does not violate $V_1 > V_1^2$ or $V_2 > V_2^f$ for small enough ϵ. However, it strictly increases the monopoly's profits. ■

Thus, with the condition $q_1 > q_f$, the problem is reduced to

$$max_{\{(q_i,T_i)\}_{i=f,1,2}} \lambda\gamma[T_f - cq_f] + \lambda(1-\gamma)[T_1 - cq_1] + (1-\lambda)[T_2 - cq_2]$$

subject to $V_1 = V_f = KV_2$ and $V_2 = V_2^1$. Call this reduced problem P_1.

Similarly, if we have $q_1 < q_f$, then the reduced problem would be

$$max_{\{(q_i,T_i)\}_{i=f,1,2}} \lambda\gamma[T_f - cq_f] + \lambda(1-\gamma)[T_1 - cq_1] + (1-\lambda)[T_2 - cq_2]$$

subject to $V_1 = V_f = KV_2$ and $V_2 = V_2^f$. Call this reduced problem P_f.

Assumption 1 guarantees that each type is served, as well as non-negative payments. Note that $\theta_1 > (1 - \lambda(1-K))\theta_2$ is equivalent to $\theta_1 > (1 - \frac{\lambda}{1+\alpha(1-\lambda)})\theta_2$. Given any λ, θ_1 and θ_2, with $\theta_2 > \theta_1 > 0$, we can actually find a threshold $\bar{\alpha} > 0$ such that $\theta_1 = (1 - \frac{\lambda}{1+\bar{\alpha}(1-\lambda)})\theta_2$. This is because $1 - \frac{\lambda}{1+\alpha(1-\lambda)}$ is strictly increasing in α. So, since $\theta_2 > \theta_1 > 0$, there exists a small enough $\bar{\alpha} > 0$ such that $\theta_1 = (1 - \frac{\lambda}{1+\bar{\alpha}(1-\lambda)})\theta_2$. So for all $0 < \alpha < \bar{\alpha}$, we have $\theta_1 > (1 - \lambda(1-K))\theta_2$. Alternatively, given any α, θ_1 and θ_2, with $\theta_2 > \theta_1 > 0$, we can find a large enough $\bar{\lambda} \in (0,1)$ such that $\theta_1 = (1 - \frac{\bar{\lambda}}{1+\alpha(1-\bar{\lambda})})\theta_2$, since $1 - \frac{\lambda}{1+\alpha(1-\lambda)}$ is strictly decreasing in λ. For all $1 > \lambda > \bar{\lambda}$, we have $\theta_1 > (1 - \lambda(1-K))\theta_2$. Note also that, a large enough λ or small enough α ensures that the low types are served in the optimal nonlinear pricing.

Case 1: Suppose $q_1 > q_f$. Then we solve P_1. Note that $V_1 = KV_2$ is equivalent to $\theta_1 u(q_1) - T_1 = K[\theta_2 u(q_2) - T_2]$. And, $V_2 = V_2^1$ is equivalent to $\theta_2 u(q_2) - T_2 = \theta_2 u(q_1) - T_1$. Solving these two equalities for T_1 and T_2, we get

$$T_1 = \theta_2 u(q_1) - \frac{\theta_2 - \theta_1}{1-K} u(q_1) = \frac{\theta_1 - K\theta_2}{1-K} u(q_1)$$

$$T_2 = \theta_2 u(q_2) - \frac{\theta_2 - \theta_1}{1-K} u(q_1) = \theta_2(u(q_2) - u(q_1)) + \frac{\theta_1 - K\theta_2}{1-K} u(q_1)$$

Note that, since $V_f = V_1$, that is, $\theta_1 u(q_f) - T_f = \theta_1 u(q_1) - T_1$, we can write $T_f = \theta_1(u(q_f) - u(q_1)) + T_1 = \theta_1(u(q_f) - u(q_1)) + \frac{\theta_1 - K\theta_2}{1-K} u(q_1)$.[26]

Now, we can insert these expressions for $\{T_i\}_{i=f,1,2}$ into the objective function and reduce the problem to an unconstrained maximization problem where the only choice variables are q_f,q_1 and q_2. That is,

$$
\begin{aligned}
max_{\{q_i\}_{i=f,1,2}} \quad & \lambda\gamma[\theta_1(u(q_f) - u(q_1)) + \frac{\theta_1 - K\theta_2}{1-K} u(q_1) - cq_f] \\
& + \lambda(1-\gamma)[\frac{\theta_1 - K\theta_2}{1-K} u(q_1) - cq_1] \\
& + (1-\lambda)[\theta_2 u(q_2) - \frac{\theta_2 - \theta_1}{1-K} u(q_1) - cq_2]
\end{aligned}
$$

Since this is a concave programming, the first order conditions will suffice. We have

$$\theta_1 u'(q_f^*) = c$$

$$\theta_2 u'(q_2^*) = c$$

and

$$\theta_1 u'(q_1^*) = \frac{\lambda(1-K)\theta_1(1-\gamma)}{\lambda(1-K)(\theta_2 - \gamma\theta_1) - (\theta_2 - \theta_1)} c$$

Lemma 7 $\theta_1 u'(q_1^*) = c' > c$.

Proof. $\theta_1 u'(q_1^*) > c$ if and only if $\frac{\lambda(1-K)\theta_1(1-\gamma)}{\lambda(1-K)(\theta_2-\gamma\theta_1)-(\theta_2-\theta_1)} > 1$ if and only if

$$\lambda(1-K)\theta_1(1-\gamma) > \lambda(1-K)(\theta_2 - \gamma\theta_1) - (\theta_2 - \theta_1)$$

if and only if

$$\theta_2 - \theta_1 > \lambda(1-K)(\theta_2 - \gamma\theta_1 - \theta_1(1-\gamma))$$

if and only if $\theta_2 - \theta_1 > \lambda(1-K)(\theta_2 - \theta_1)$ if and only if $1 > \lambda(1-K)$. Since $K = \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)}$, this last condition is equivalent to $1 > \lambda(1 - \frac{\alpha(1-\lambda)}{1+\alpha(1-\lambda)}) = \frac{\lambda}{1+\alpha(1-\lambda)}$, which is equivalent to $1+\alpha(1-\lambda) > \lambda$. This can be written as $1 > \lambda - \alpha(1-\lambda) = \lambda - \alpha + \alpha\lambda = \lambda(1+\alpha) - \alpha$. Thus, we get $1+\alpha > \lambda(1+\alpha)$, which is always satisfied since $1 > \lambda$. ■

However, the quantity levels q_f^* and q_1^* are such that $q_f^* > q_1^*$, contradicting the supposition for this Case 1. To see $q_f^* > q_1^*$, note that $\theta_1 u'(q_f^*) = c < \theta_1 u'(q_1^*)$. Thus, $u'(q_f^*) < u'(q_1^*)$, which implies $q_f^* > q_1^*$ since $u'' < 0$.

[26]Note that, $\theta_1 \geq (1-\lambda(1-K))\theta_2$ implies $\theta_1 \geq K\theta_2$, which in turn implies $T_2 \geq T_1 \geq 0$.

Case 2: Suppose $q_1 < q_f$. Then we solve P_f. Note that $V_f = KV_2$ is equivalent to $\theta_1 u(q_f) - T_f = K[\theta_2 u(q_2) - T_2]$. And, $V_2 = V_2^f$ is equivalent to $\theta_2 u(q_2) - T_2 = \theta_2 u(q_f) - T_f$. Solving these two equalities for T_f and T_2, we get

$$T_f = \theta_2 u(q_f) - \frac{\theta_2 - \theta_1}{1-K} u(q_f) = \frac{\theta_1 - K\theta_2}{1-K} u(q_f)$$

$$T_2 = \theta_2 u(q_2) - \frac{\theta_2 - \theta_1}{1-K} u(q_f) = \theta_2 (u(q_2) - u(q_f)) + \frac{\theta_1 - K\theta_2}{1-K} u(q_f)$$

Note that, since $V_f = V_1$, that is, $\theta_1 u(q_f) - T_f = \theta_1 u(q_1) - T_1$, we can write $T_1 = \theta_1 (u(q_1) - u(q_f)) + T_f = \theta_1 (u(q_1) - u(q_f)) + \frac{\theta_1 - K\theta_2}{1-K} u(q_f)$.[27]

Now, we can insert these expressions for $\{T_i\}_{i=f,1,2}$ into the objective function and reduce the problem to an unconstrained maximization problem where the only choice variables are q_f,q_1 and q_2. That is,

$$max_{\{q_i\}_{i=f,1,2}} \quad \lambda\gamma[\frac{\theta_1 - K\theta_2}{1-K} u(q_f) - cq_f]$$

$$+\lambda(1-\gamma)[\theta_1 (u(q_1) - u(q_f)) + \frac{\theta_1 - K\theta_2}{1-K} u(q_f) - cq_1]$$

$$+(1-\lambda)[\theta_2 u(q_2) - \frac{\theta_2 - \theta_1}{1-K} u(q_f) - cq_2]$$

Since this is a concave programming, the first order conditions will suffice. We have

$$\theta_1 u'(q_f^*) = \frac{\lambda(1-K)\theta_1(1-\gamma)}{\lambda(1-K)(\theta_2 - \gamma\theta_1) - (\theta_2 - \theta_1)} c$$

$$\theta_2 u'(q_2^*) = c$$

$$\theta_1 u'(q_1^*) = c$$

Lemma 8 $\theta_1 u'(q_f^*) > c$.

Proof. Follows from $\frac{\lambda(1-K)\theta_1(1-\gamma)}{\lambda(1-K)(\theta_2-\gamma\theta_1)-(\theta_2-\theta_1)} > 1$ which was shown in the proof of Lemma 7. ■

However, the quantity levels q_f^* and q_1^* are such that $q_1^* > q_f^*$, contradicting the supposition for this Case 2. To see $q_1^* > q_f^*$, note that $\theta_1 u'(q_1^*) = c < \theta_1 u'(q_f^*)$. Thus, $u'(q_1^*) < u'(q_f^*)$, which implies $q_1^* > q_f^*$ since $u'' < 0$.

Thus, Case 1 and Case 2 together imply the following lemma:

Lemma 9 $q_f^* = q_1^*$ *and* $T_f^* = T_1^*$.

[27]Note that, if $\theta_1 \geq K\theta_2$, then $T_2 \geq T_f \geq 0$.

Proof. $q_f^* = q_1^*$ follows from the analysis conducted in Case 1 and Case 2. $T_f^* = T_1^*$ follows from $q_f^* = q_1^*$ and $V_1 = V_f$. ■

Now, in the light of Lemma 9, the reduced problem is

$$max_{\{(q_i,T_i)\}_{i=1,2}} \lambda\gamma[T_1 - cq_1] + \lambda(1-\gamma)[T_1 - cq_1] + (1-\lambda)[T_2 - cq_2]$$

subject to $T_1 = \frac{\theta_1 - K\theta_2}{1-K}u(q_1)$ and $T_2 = \theta_2(u(q_2) - u(q_1)) + T_1$. Inserting T_1 and T_2, we get

$$max_{\{q_i\}_{i=1,2}} \quad \lambda[\frac{\theta_1 - K\theta_2}{1-K}u(q_1) - cq_1] + (1-\lambda)[\theta_2(u(q_2) - u(q_1)) + \frac{\theta_1 - K\theta_2}{1-K}u(q_1) - cq_2]$$

or

$$max_{\{q_1,q_2\}} \quad \frac{\theta_1 - K\theta_2}{1-K}u(q_1) + (1-\lambda)\theta_2(u(q_2) - u(q_1)) - \lambda cq_1 - (1-\lambda)cq_2$$

First order conditions give

$$\theta_1 u'(q_f^*) = \theta_1 u'(q_1^*) = \frac{\lambda(1-K)\theta_1}{\lambda(1-K)\theta_2 - (\theta_2 - \theta_1)}c > c$$

$$\theta_2 u'(q_2^*) = c$$

Note that $\frac{\lambda(1-K)\theta_1}{\lambda(1-K)\theta_2 - (\theta_2-\theta_1)} > 1$ is a special case of the proof of Lemma 7, with $\gamma = 0$. Thus, it immediately follows. Also, note that the denominator is always non-negative under Assumption 1, since $\theta_1 > (1 - \lambda(1 - K))\theta_2$ implies $\lambda(1-K)\theta_2 > \theta_2 - \theta_1$.

Lemma 10 $q_2^* > q_1^* = q_f^*$.

Proof. We have $\theta_1 u'(q_f^*) = \theta_1 u'(q_f^*) > c = \theta_2 u'(q_2^*)$ where $\theta_2 > \theta_1$. Thus, $u'(q_f^*) = u'(q_1^*) > u'(q_2^*)$. Since $u'' < 0$, we get $q_2^* > q_f^* = q_1^*$. ■

Lemma 11 $T_2^* > T_f^* = T_1^*$.

Proof. $T_f^* = T_1^*$ directly follows from $q_f^* = q_1^*$ and $V_1 = V_f$. And, $T_2^* = \theta_2(u(q_2^*) - u(q_1^*)) + T_1^* > T_1^*$ since $q_2^* > q_1^*$ and $u' > 0$. Thus, $T_2^* > T_f^* > T_1^*$. ■

Finally, we need to check $V_1 \geq V_1^2$ constraint at the optimal levels, as well as $V_2 > V_1 = V_f$.

Lemma 12 $V_1 > V_1^2$ *at* $\{(q_i^*, T_i^*)\}_{i=f,1,2}$.

Proof. $V_1 > V_1^2$ if and only if $\theta_1 u(q_1^*) - T_1^* > \theta_1 u(q_2^*) - T_2^*$ if and only if $T_2^* - T_1^* > \theta_1(u(q_2^*) - u(q_1^*))$ if and only if $\theta_2(u(q_2^*) - u(q_1^*)) > \theta_1(u(q_2^*) - u(q_1^*))$ if and only if $\theta_2 > \theta_1$, which holds. ■

Lemma 13 $V_2 > V_1 = V_f$ *at* $\{(q_i^*, T_i^*)\}_{i=f,1,2}$.

Proof. $V_2 = \theta_2 u(q_2^*) - T_2^* = \theta_2 u(q_2^*) - \theta_2(u(q_2^*) - u(q_1^*)) - T_1^* = \theta_2 u(q_1^*) - T_1^* > \theta_1 u(q_1^*) - T_1^* = V_1$ since $\theta_2 > \theta_1$. ■

This finishes the proof of Propostion 1. ■

7.2 Appendix B

Proof of Lemma 1. We prove Lemma 1 through a series of lemmas, Lemma 14 through 24.

Lemma 14 *If* $\theta' > \theta$ *then* $V(q_n(\theta'), \theta') > V(q_n(\theta), \theta)$.

Proof. $V(q_n(\theta'), \theta') = \max_q \theta' u(q) - T(q) \geq \theta' u(q_n(\theta)) - T(q_n(\theta)) > \theta u(q_n(\theta)) - T(q_n(\theta)) = V(q_n(\theta), \theta)$ ■

Lemma 15 *If* $\theta' > \theta$ *then* $W(q_f(\theta'), \theta') > W(q_f(\theta), \theta)$.

Proof. A fair type with the demand type θ' has the following utility.

$$W(q_f(\theta'), \theta') = \max_q V(q, \theta') - \alpha[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{V(q_f(\hat{\theta}), \hat{\theta}) - V(q, \theta'), 0\} f(\hat{\theta}) d\hat{\theta}$$

$$+(1-\gamma) \int_{\underline{\theta}}^{\bar{\theta}} \max\{V(q_n(\hat{\theta}), \hat{\theta}) - V(q, \theta'), 0\} f(\hat{\theta}) d\hat{\theta}]$$

By $IC^{f,f}$ we have $W(q_f(\theta'), \theta') \geq W(q_f(\theta), \theta')$ for any other θ. Thus, we have

$$W(q_f(\theta'), \theta') \geq V(q_f(\theta), \theta') - \alpha[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{V(q_f(\hat{\theta}), \hat{\theta}) - V(q_f(\theta), \theta'), 0\} f(\hat{\theta}) d\hat{\theta}$$

$$+(1-\gamma) \int_{\underline{\theta}}^{\bar{\theta}} \max\{V(q_n(\hat{\theta}), \hat{\theta}) - V(q_f(\theta), \theta'), 0\} f(\hat{\theta}) d\hat{\theta}]$$

$$\geq V(q_f(\theta), \theta) - \alpha[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{V(q_f(\hat{\theta}), \hat{\theta}) - V(q_f(\theta), \theta), 0\} f(\hat{\theta}) d\hat{\theta}$$

$$+(1-\gamma) \int_{\underline{\theta}}^{\bar{\theta}} \max\{V(q_n(\hat{\theta}), \hat{\theta}) - V(q_f(\theta), \theta), 0\} f(\hat{\theta}) d\hat{\theta}]$$

$$= W(q_f(\theta), \theta)$$

where the second inequality above follows from the fact that $V(q, \theta') \geq V(q, \theta)$ when $\theta' > \theta$, since $V(q, \theta') = \theta' u(q) - T(q) \geq \theta u(q) - T(q) = V(q, \theta)$.[28] Thus, we have $W(q_f(\theta'), \theta') \geq W(q_f(\theta), \theta)$ when $\theta' > \theta$. ■

[28]Note that this observation is slightly different from what we have in Lemma 14.

Lemma 16 *For all θ, $V(q_n(\theta),\theta) \geq V(q_f(\theta),\theta)$.*

Proof. By $IC^{n,f}(\theta)$, $V(q_n(\theta),\theta) = \max_q \theta u(q) - T(q) \geq \theta u(q_f(\theta)) - T(q_f(\theta)) = V(q_f(\theta),\theta)$. ■

Lemma 17 *If $\theta' > \theta$, then $V(q_f(\theta'),\theta') > V(q_f(\theta),\theta)$.*

Proof. $W(V) = V - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}),\hat{\theta}) - V\} f(\hat{\theta})d\hat{\theta} + (1-\gamma) \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}),\hat{\theta}) - V\} f(\hat{\theta})d\hat{\theta}\Big] \geq 0$ is increasing in V. Then, by Lemma 15 we have $V(q_f(\theta'),\theta') > V(q_f(\theta),\theta)$ for $\theta' > \theta$. ■

Lemma 18 *Under Assumption 2, $q_n(\theta)$ is strictly increasing in θ.*

Proof. Suppose $\hat{\theta} > \theta$. Then, from the first order conditions of the optimizations of $\hat{\theta}$ type and θ type neutral buyers, we have $\hat{\theta}u'(q_n(\hat{\theta})) - T'(q_n(\hat{\theta})) = 0$ and $\theta u'(q_n(\theta)) - T'(q_n(\theta)) = 0$. Then, $\hat{\theta}u'(q_n(\theta)) - T'(q_n(\theta)) > 0$ since $\hat{\theta} > \theta$ and $u'(\cdot) > 0$. Thus, $V'(q_n(\hat{\theta}),\hat{\theta}) = \hat{\theta}u'(q_n(\hat{\theta})) - T'(q_n(\hat{\theta})) = 0$ and $V'(q_n(\theta),\hat{\theta}) = \hat{\theta}u'(q_n(\theta)) - T'(q_n(\theta)) > 0$ together imply $q_n(\hat{\theta}) > q_n(\theta)$, since $V(q,\theta)$ is strictly concave in q. ■

Lemma 19 *For any θ, $IR^n(\theta)$ is implied by $IR^n(\underline{\theta})$.*

Proof. By Lemma 14, we have $V(q_n(\theta),\theta) > V(q_n(\underline{\theta}),\underline{\theta})$ since $\theta > \underline{\theta}$. When $IR^n(\underline{\theta})$ holds we have $V(q_n(\underline{\theta}),\underline{\theta}) \geq 0$. Thus, $V(q_n(\theta),\theta) > 0$, that is $IR^n(\theta)$ holds. ■

Lemma 20 *For any θ, $IR^f(\theta)$ is implied by $IR^f(\underline{\theta})$.*

Proof. By Lemma 15, we have $W(q_f(\theta),\theta) > W(q_f(\underline{\theta}),\underline{\theta})$ since $\theta > \underline{\theta}$. When $IR^f(\underline{\theta})$ holds we have $W(q_f(\underline{\theta}),\underline{\theta}) \geq 0$. Thus, $W(q_f(\theta),\theta) > 0$, that is $IR^f(\theta)$ holds. ■

Lemma 21 *If $IR^f(\underline{\theta})$ holds, then all IR constraints hold.*

Proof. When $IR^f(\underline{\theta})$ holds, all $IR^f(\theta)$ hold for any θ by Lemma 20. And since $IR^f(\underline{\theta})$ holds, we have

$$W(q_f(\underline{\theta}),\underline{\theta}) = V(q_f(\underline{\theta}),\underline{\theta}) - \alpha[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{(V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\underline{\theta},\underline{\theta})),0\} f(\hat{\theta})d\hat{\theta}$$

$$+(1-\gamma) \int_{\underline{\theta}}^{\bar{\theta}} \max\{(V(q_n(\hat{\theta}),\hat{\theta}) - V(q_f(\underline{\theta}),\underline{\theta})),0\} f(\hat{\theta})d\hat{\theta}] \geq 0$$

Note that $V(q_n(\underline{\theta}),\underline{\theta}) \geq V(q_f(\underline{\theta}),\underline{\theta}) \geq W(q_f(\underline{\theta}),\underline{\theta}) \geq 0$, where the first inequality follows from Lemma 16 and the second inequality follows from

$$\alpha[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{(V(q_f(\hat{\theta}),\hat{\theta}) - V(q_f(\underline{\theta}),\underline{\theta})),0\} f(\hat{\theta})d\hat{\theta} + (1-\gamma) \int_{\underline{\theta}}^{\bar{\theta}} \max\{(V(q_n(\hat{\theta}),\hat{\theta}) - V(q_f(\underline{\theta}),\underline{\theta})),0\} f(\hat{\theta})d\hat{\theta}] \geq 0$$

Thus, $V(q_n(\underline{\theta}), \underline{\theta}) \geq 0$, that is $IR^n(\underline{\theta})$ holds. By Lemma 19, all $IR^n(\theta)$ also hold for any θ. Thus, all IR constraints hold when, $IR^f(\underline{\theta})$. ■

Lemma 22 *For all θ, $V(q_n(\theta), \theta) = V(q_f(\theta), \theta)$.*

Proof. For any given θ, $IC^{n,f}(\theta)$ implies that $V(q_n(\theta), \theta) \geq V(q_f(\theta), \theta)$. Also, $IC^{f,n}(\theta)$ implies $W(V(q_f(\theta), \theta)) \geq W(V(q_n(\theta), \theta))$, where

$$W(V) = V - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}), \hat{\theta}) - V\} f(\hat{\theta}) d\hat{\theta} + (1-\gamma) \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}), \hat{\theta}) - V\} f(\hat{\theta}) d\hat{\theta}\Big]$$

Since $W(V)$ is increasing in V, we have $V(q_f(\theta), \theta) \geq V(q_n(\theta), \theta)$ for all θ. Thus, for all θ we have $V(q_n(\theta), \theta) = V(q_f(\theta), \theta)$. ■

Lemma 23 *Under Assumption 2, $q_f(\theta) = q_n(\theta)$ for all θ.*

Proof. For any given θ, $V(q_n(\theta), \theta) = V(q_f(\theta), \theta)$ by Lemma 22. By Assumption 2, solution to $V'(q, \theta) = 0$ is unique, thus, $q_f(\theta) = q_n(\theta)$ ■

Lemma 24 *Under Assumption 2, $IC^{n,n}(\theta)$ implies $IC^{n,f}(\theta)$, $IC^{f,f}(\theta)$ and $IC^{f,n}(\theta)$, for all θ.*

Proof. $IC^{n,f}(\theta)$ holds since $\theta u(q_n(\theta)) - T(q_n(\theta)) \geq \theta u(q_n(\hat{\theta})) - T(q_n(\hat{\theta})) = \theta u(q_f(\hat{\theta})) - T(q_f(\hat{\theta}))$ for all $\hat{\theta}$, where the first inequality is by $IC^{n,n}(\theta)$ and the second inequality is by Lemma 23. To see that $IC^{f,f}(\theta)$ holds, note that by $IC^{n,n}(\theta)$, we have $V(q_n(\theta), \theta) \geq V(q_n(\hat{\theta}), \theta)$. Since $W(V)$ is increasing in V, we have $W(V(q_n(\theta), \theta)) \geq W(V(q_n(\hat{\theta}), \theta))$. By Lemma 23, we have $q_f(\theta) = q_n(\theta)$ and $q_f(\hat{\theta}) = q_n(\hat{\theta})$. Thus, $W(V(q_f(\theta), \theta)) \geq W(V(q_f(\hat{\theta}), \theta))$ for all $\hat{\theta}$. $IC^{f,n}(\theta)$ follows from $IC^{f,f}(\theta)$ and $q_f(\hat{\theta}) = q_n(\hat{\theta})$. ■

Combining Lemma 21 and 24, we reduce the monopoly's problem to the following problem.

$$max_{\{q_t(\theta), T_t(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}], t \in \{f,n\}}} \int_{\underline{\theta}}^{\bar{\theta}} [\gamma [T_f(\theta) - c q_f(\theta)] + (1-\gamma)[T_n(\theta) - c q_n(\theta)]] f(\theta) d\theta$$

subject to

$IR^f(\underline{\theta}) : W(\underline{\theta}) \geq 0$ and

$IC^{n,n}(\theta) : \theta u'(q_n(\theta)) - T'(q_n(\theta)) = 0$ for all θ.

Note that $IR^f(\underline{\theta})$ boils down to $V(q_f(\underline{\theta}), \underline{\theta}) - \alpha[\int_{\underline{\theta}}^{\bar{\theta}} (V(q_f(\hat{\theta}), \hat{\theta}) - V(q_f(\underline{\theta}), \underline{\theta})) f(\hat{\theta}) d\hat{\theta}] \geq 0$. Also note that this $IR^f(\underline{\theta})$ must bind, otherwise the monopoly could keep the quantities the same and ask for a larger payment without violating any other constraint. Thus, we must have $W(\underline{\theta}) = 0$.

Again by Lemma 23, $IC^{n,n}(\theta)$ is equivalent to $\theta u'(q_f(\theta)) - T'(q_f(\theta)) = 0$ for all θ. Also, the objective function boils down to one below. Thus, we have the reduced problem as follows

$$max_{\{q_f(\theta),T(q_f(\theta))\}_{\theta\in[\underline{\theta},\bar{\theta}]}} \int_{\underline{\theta}}^{\bar{\theta}} [T(q_f(\theta)) - cq_f(\theta)]f(\theta)d\theta$$

subject to

$IR^f(\underline{\theta}) : V(q_f(\underline{\theta}), \underline{\theta}) - \alpha[\int_{\underline{\theta}}^{\bar{\theta}} (V(q_f(\hat{\theta}), \hat{\theta}) - V(q_f(\underline{\theta}), \underline{\theta}))f(\hat{\theta})d\hat{\theta}] = 0$ and

$IC^{n,n}(\theta) : \theta u'(q_f(\theta)) - T'(q_f(\theta)) = 0$ for all θ. which finishes the proof of Lemma 1. ■

7.3 Appendix C

Lemma 25 $\tilde{\theta}_f \geq \tilde{\theta}_n$.

Proof. Suppose $\tilde{\theta}_n > \tilde{\theta}_f$. Then, by $IR^f(\tilde{\theta}_f)$, we have $W(q(\tilde{\theta}_f), \tilde{\theta}_f) \geq 0$. Note that $W(V) = V - \alpha\Big[\gamma \int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}), \hat{\theta}) - V\}f(\hat{\theta})d\hat{\theta} + (1-\gamma)\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_n(\hat{\theta}), \hat{\theta}) - V\}f(\hat{\theta})d\hat{\theta}\Big] \geq 0$ is increasing in V. Then, by Lemma 15 we have $V(q_f(\theta'), \theta') > V(q_f(\theta), \theta)$ for $\theta' > \theta$. Thus, $\int_{\underline{\theta}}^{\bar{\theta}} \max\{0, V(q_f(\hat{\theta}), \hat{\theta}) - V\}f(\hat{\theta})d\hat{\theta} > 0$, that is, $V(q(\tilde{\theta}_f), \tilde{\theta}_f) > 0$. Therefore, neutral type with taste parameter $\tilde{\theta}_f$ can mimic fair type and purchase $q(\tilde{\theta}_f)$ to get positive net utility rather than being excluded and getting zero. Hence, it must be $\tilde{\theta}_f \geq \tilde{\theta}_n$ in the optimal nonlinear pricing. ■

By Lemma 25, the monopoly sells only to neutral types for the interval $[\tilde{\theta}_n, \tilde{\theta}_f]$ and to both types for higher values of θ. Thus, the monopoly profit becomes

$$\Pi(\tilde{\theta}_f, \tilde{\theta}_n) = (1-\gamma)\int_{\tilde{\theta}_n}^{\bar{\theta}} \Big(T(q_n(\theta)) - cq_n(\theta)\Big)f(\theta)d\theta + \gamma \int_{\tilde{\theta}_f}^{\bar{\theta}} \Big(T(q_f(\theta)) - cq_f(\theta)\Big)f(\theta)d\theta$$

The following Lemma provides a reduced problem equivalent to the monopoly's original problem.

Lemma 26 *The monopoly's problem reduces to* $max_{q_n,q_f}\Pi(\tilde{\theta}_f, \tilde{\theta}_n)$ *subject to*

$IR^n(\tilde{\theta}_n) : V(q(\tilde{\theta}_n), \tilde{\theta}_n) = 0,$

$IR^f(\tilde{\theta}_f) : V(q(\tilde{\theta}_f), \tilde{\theta}_f) - \alpha[\int_{\tilde{\theta}_f}^{\bar{\theta}} (V(q(\hat{\theta}), \hat{\theta}) - V(q(\tilde{\theta}_f), \tilde{\theta}_f))f(\hat{\theta})d\hat{\theta}] = 0,$

$IC^{n,n}(\theta) : \theta u'(q(\theta)) - T'(q(\theta)) = 0$ *for all* $\theta \in [\tilde{\theta}_n, \tilde{\theta}_f]$,

$IC^{n,n}(\theta) : \theta u'(q(\theta)) - T'(q(\theta)) = 0$ *for all* $\theta \in [\tilde{\theta}_f, \bar{\theta}]$ *and*

$\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta))d\theta = \alpha \int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1 - F(\theta))d\theta$

Proof. We prove by a series of lemmas, Lemma 27 through 30, similar to those in Appendix 7.2.

Lemma 27 *For any* $\theta \geq \tilde{\theta}_n$, $IR^n(\theta)$ *is implied by* $IR^n(\tilde{\theta}_n)$.

Proof. The proof is similar to the proof of Lemma 19, where we use $V(q_n(\theta'), \theta') > V(q_n(\theta), \theta)$ for all $\theta', \theta \geq \tilde{\theta}_n$ with $\theta' > \theta$, which is proven by a similar argument in the proof of Lemma 14. ■

Lemma 28 *For any $\theta \geq \tilde{\theta}_f$, $IR^f(\theta)$ is implied by $IR^f(\tilde{\theta}_f)$.*

Proof. The proof is similar to the proof of Lemma 20, where we use $W(q_f(\theta'), \theta') > W(q_f(\theta), \theta)$ for all $\theta', \theta \geq \tilde{\theta}_n$ with $\theta' > \theta$, which is proven by a similar argument in the proof of Lemma 15. ■

Lemma 29 *Under Assumption 2, $q_f(\theta) = q_n(\theta)$ for all $\theta \geq \tilde{\theta}_f$.*

Proof. Using the proof of Lemma 22, we get, for all $\theta \geq \tilde{\theta}_f$, $V(q_n(\theta), \theta) = V(q_f(\theta), \theta)$. Then, by a similar proof of Lemma 23, we get $q_f(\theta) = q_n(\theta)$ for all $\theta \geq \tilde{\theta}_f$. ■

Lemma 30 *Under Assumption 2, all IC constraints are implied by the following three constraints*
(i) $IC^{n,n}(\theta)$ for all $\theta \in [\tilde{\theta}_n, \tilde{\theta}_f]$,
(ii) $IC^{n,n}(\theta)$ for all $\theta \in [\tilde{\theta}_f, \bar{\theta}]$ and
(iii) $\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta))d\theta = \alpha \int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1 - F(\theta))d\theta$

Proof. First, we need to show that $IC^{n,n}(\theta)$ for all $\theta \in [\tilde{\theta}_n, \bar{\theta}]$ is implied by these three constraints. Take $\theta_1 \in [\tilde{\theta}_n, \tilde{\theta}_f]$ and $\theta_2 \in [\tilde{\theta}_f, \bar{\theta}]$. Constraint (i) implies that $V(\theta_1, q_n(\theta_1)) \geq V(\theta_1, q_n(\tilde{\theta}_f))$ which is equivalent to

$$\theta_1 u(q_n(\theta_1)) - \theta_1 u(q_n(\tilde{\theta}_f)) \geq T(q_n(\theta_1)) - T(q_n(\tilde{\theta}_f))$$

Constraint (ii) implies that $V(\tilde{\theta}_f, q_f(\tilde{\theta}_f)) \geq V(\tilde{\theta}_f, q_f(\theta_2))$. By constraint (iii) we know that $V(\tilde{\theta}_f, q_f(\tilde{\theta}_f)) = V(\tilde{\theta}_f, q_n(\tilde{\theta}_f))$. Hence, the implication of constraint (ii) becomes

$$\tilde{\theta}_f u(q_n(\tilde{\theta}_f)) - \tilde{\theta}_f u(q_f(\theta_2)) \geq T(q_n(\tilde{\theta}_f)) - T(q_f(\theta_2))$$

By adding up two inequalities, we get

$$\theta_1 u(q_n(\theta_1)) - \theta_1 u(q_n(\theta_2)) + (\tilde{\theta}_f - \theta_1)(u(q_n(\tilde{\theta}_f)) - u(q_f(\theta_2))) \geq T(q_n(\theta_1)) - T(q_f(\theta_2))$$

By Lemma 29, $q_f(\theta) = q_n(\theta)$ for all $\theta \geq \tilde{\theta}_f$. Since $q_f(\theta)$ and $u(q)$ are increasing functions, $u(q_n(\tilde{\theta}_f)) - u(q_f(\theta_2))$ is negative and we have

$$(\tilde{\theta}_f - \theta_1)(u(q_n(\tilde{\theta}_f)) - u(q_f(\theta_2))) \leq 0$$

This implies $IC^{n,n}(\theta_1)$ for θ_2:

$$\theta_1 u(q_n(\theta_1)) - \theta_1 u(q_n(\theta_2)) \geq T(q_n(\theta_1)) - T(q_f(\theta_2))$$

We can prove $IC^{n,n}(\theta_1)$ for θ_2 by the same reasoning. We showed that $IC^{n,n}(\theta)$ for all $\theta \in [\tilde{\theta}_n, \bar{\theta}]$ is implied by these three constraints. Then, we can apply the proof of Lemma 24. ■

This finishes the proof of Lemma 26. ■

The next result characterizes the optimal quantities and the thresholds, $\tilde{\theta}_n$ and $\tilde{\theta}_f$.

Lemma 31 *In the optimal nonlinear pricing when exclusion is allowed, the quantities should be* $q_n^*(\theta)$ *and* $q_f^*(\theta)$ *with* $q_n^*(\theta) = q_f^*(\theta)$ *for all* $\theta \in [\tilde{\theta}_f, \bar{\theta}]$*, where*
(i) $q_n^*(\theta)$ *solves* $u'(q(\theta))[\theta + \frac{F(\theta)-F(\tilde{\theta}_n)}{f(\theta)}] = c$*, in* $[\tilde{\theta}_n, \tilde{\theta}_f]$.
(ii) $q_f^*(\theta)$ *solves* $u'(q(\theta))[\theta - H(\theta) - \alpha(1-F(\tilde{\theta}_f))H(\theta) - (1-\gamma)(F(\tilde{\theta}_f) - F(\tilde{\theta}_n))\alpha H(\theta)] = c$*, in* $[\tilde{\theta}_f, \bar{\theta}]$.
and thresholds $\tilde{\theta}_n$ *and* $\tilde{\theta}_f$ *solve*
(iii) $\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta))f(\tilde{\theta}_n)d\theta + \int_{\tilde{\theta}_f}^{\bar{\theta}} f(\tilde{\theta}_n)\alpha u(q_f(\theta))(1-F(\theta))d\theta - \Big(\theta u(q_n(\theta)) + u(q_n(\theta))\frac{F(\theta)-F(\tilde{\theta}_n)}{f(\theta)} - cq_n(\theta)\Big)f(\theta) = 0$
(iv) $\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta))d\theta = \alpha \int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1-F(\theta))d\theta$

Proof. First, we solve quantities for a given $\tilde{\theta}_n$, then we maximize over $\tilde{\theta}_n$. Also note that $q_n^*(\theta) = q_f^*(\theta)$ for all $\theta \in [\tilde{\theta}_f, \bar{\theta}]$ follows directly from Lemma 29. This implies

$$\Pi(\tilde{\theta}_f, \tilde{\theta}_n) = (1-\gamma)\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} \Big(T(q_n(\theta)) - cq_n(\theta)\Big)f(\theta)d\theta + \int_{\tilde{\theta}_f}^{\bar{\theta}} \Big(T(q_f(\theta)) - cq_f(\theta)\Big)f(\theta)d\theta$$

where

$$T(q_n(\theta)) = \theta u(q_n(\theta)) - V(q_n(\theta),\theta) = \theta u(q_n(\theta)) - [\int_{\tilde{\theta}_n}^{\theta} u(q_n(\hat{\theta}))d\hat{\theta} + V(q_n(\tilde{\theta}_n), \tilde{\theta}_n)]$$

Since $V(q_n(\tilde{\theta}_n), \tilde{\theta}_n) = 0$ by binding $IR^n(\tilde{\theta}_n)$, we have

$$T(q_n(\theta)) = \theta u(q_n(\theta)) - V(q_n(\theta),\theta) = \theta u(q_n(\theta)) - \int_{\tilde{\theta}_n}^{\theta} u(q_n(\hat{\theta}))d\hat{\theta}$$

By the last condition in Lemma 26, at $\tilde{\theta}_f$, we have

$$\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta))d\theta = \alpha \int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1-F(\theta))d\theta$$

which implies

$$T(q_n(\theta)) = \theta u(q_n(\theta)) - [\alpha \int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1-F(\theta))d\theta - \int_{\theta}^{\tilde{\theta}_f} u(q_n(\hat{\theta}))d\hat{\theta}]$$

By $IR^f(\tilde{\theta}_f)$ we have $W(q_f^*(\tilde{\theta}_f), \tilde{\theta}_f) = 0$, that is

$$V(q(\tilde{\theta}_f), \tilde{\theta}_f) - \alpha[\int_{\tilde{\theta}_f}^{\bar{\theta}} (V(q(\hat{\theta}), \hat{\theta}) - V(q(\tilde{\theta}_f), \tilde{\theta}_f)) f(\hat{\theta}) d\hat{\theta}] = 0$$

We also have

$$V(q(\theta), \theta) = \int_{\tilde{\theta}_f}^{\theta} u(q(\hat{\theta})) d\hat{\theta} + V(q(\tilde{\theta}_f), \tilde{\theta}_f)$$

These two equations imply

$$T(q_f(\theta)) = \theta u(q_f(\theta)) - \Big[\int_{\tilde{\theta}_f}^{\theta} u(q_f(\hat{\theta})) d\hat{\theta} + V(q(\tilde{\theta}_f), \tilde{\theta}_f) \Big]$$

that is,

$$T(q_f(\theta)) = \theta u(q_f(\theta)) - \int_{\tilde{\theta}_f}^{\theta} u(q_f(\hat{\theta})) d\hat{\theta} - \alpha \int_{\tilde{\theta}_f}^{\bar{\theta}} \Big(\int_{\tilde{\theta}_f}^{\hat{\theta}} u(q(\theta')) d\theta' \Big) f(\hat{\theta}) d\hat{\theta}$$

By integration by parts we get

$$T(q_f(\theta)) = \theta u(q_f(\theta)) - \int_{\tilde{\theta}_f}^{\theta} u(q_f(\hat{\theta})) d\hat{\theta} - \alpha[\int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\hat{\theta}))(1 - F(\hat{\theta})) d\hat{\theta}]$$

Thus, the profit becomes

$$\begin{aligned}
\Pi(\tilde{\theta}_f, \tilde{\theta}_n) &= (1-\gamma) \int_{\tilde{\theta}_n}^{\tilde{\theta}_f} \left(\theta u(q_n(\theta)) - [\alpha \int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1 - F(\theta)) d\theta - \int_{\theta}^{\tilde{\theta}_f} u(q_n(\hat{\theta})) d\hat{\theta}] - c q_n(\theta) \right) f(\theta) d\theta \\
&+ \int_{\tilde{\theta}_f}^{\bar{\theta}} \left(\theta u(q_f(\theta)) - \int_{\tilde{\theta}_f}^{\theta} u(q_f(\hat{\theta})) d\hat{\theta} - \alpha[\int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1 - F(\theta)) d\theta] - c q_f(\theta) \right) f(\theta) d\theta
\end{aligned}$$

Integrating by parts gives

$$\begin{aligned}
\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} \int_{\theta}^{\tilde{\theta}_f} u(q_n(\hat{\theta})) d\hat{\theta} f(\theta) d\theta &= \Big(F(\theta) \int_{\theta}^{\tilde{\theta}_f} u(q_n(\hat{\theta})) d\hat{\theta} \Big) \Big|_{\tilde{\theta}_n}^{\tilde{\theta}_f} - \int_{\tilde{\theta}_n}^{\tilde{\theta}_f} -u(q_n(\theta)) F(\theta) d\theta \\
&= -F(\tilde{\theta}_n) \int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta)) d\theta + \int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta)) F(\theta) d\theta \\
&= \int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta))(F(\theta) - F(\tilde{\theta}_n)) d\theta
\end{aligned}$$

Since $\int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1-F(\theta))d\theta$ is a constant in $\Pi(\tilde{\theta}_f, \tilde{\theta}_n)$, we can take it out of the integral and get

$$\begin{aligned}
\Pi(\tilde{\theta}_f, \tilde{\theta}_n) &= (1-\gamma)(F(\tilde{\theta}_n) - F(\tilde{\theta}_f))\alpha \int_{\tilde{\theta}_f}^{\bar{\theta}} u(q_f(\theta))(1-F(\theta))d\theta \\
&+ (1-\gamma)\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} \left(\theta u(q_n(\theta)) + u(q_n(\theta))\frac{F(\theta)-F(\tilde{\theta}_n)}{f(\theta)} - cq_n(\theta)\right) f(\theta)d\theta \\
&+ \int_{\tilde{\theta}_f}^{\bar{\theta}} \left(\theta u(q_f(\theta)) - u(q_f(\theta))\frac{(1-F(\theta))}{f(\theta)} - \alpha u(q_f(\theta))(1-F(\tilde{\theta}_f))\frac{(1-F(\theta))}{f(\theta)} - cq_f(\theta)\right) f(\theta)d\theta
\end{aligned}$$

By rearranging we get,

$$\begin{aligned}
\Pi(\tilde{\theta}_f, \tilde{\theta}_n) &= (1-\gamma)\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} \left(\theta u(q_n(\theta)) + u(q_n(\theta))\frac{F(\theta)-F(\tilde{\theta}_n)}{f(\theta)} - cq_n(\theta)\right) f(\theta)d\theta \\
&+ \int_{\tilde{\theta}_f}^{\bar{\theta}} \Bigg(\theta u(q_f(\theta)) - u(q_f(\theta))\frac{(1-F(\theta))}{f(\theta)} - \alpha u(q_f(\theta))(1-F(\tilde{\theta}_f))\frac{(1-F(\theta))}{f(\theta)} \\
&- (1-\gamma)(F(\tilde{\theta}_f) - F(\tilde{\theta}_n))\alpha u(q_f(\theta))\frac{(1-F(\theta))}{f(\theta)} - cq_f(\theta)\Bigg) f(\theta)d\theta
\end{aligned}$$

Similar to previous section, we should maximize pointwise. Thus, we have two equations for two different intervals, which together characterize the optimal quantities and a FOC for optimal $\tilde{\theta}_n$.

For $\theta \in [\tilde{\theta}_n, \tilde{\theta}_f]$, we have

$$u'(q(\theta))[\theta + \frac{F(\theta)-F(\tilde{\theta}_n)}{f(\theta)}] = c \tag{4}$$

For $\theta \in [\tilde{\theta}_f, \bar{\theta}]$, we have

$$u'(q(\theta))[\theta - H(\theta) - \alpha(1-F(\tilde{\theta}_f))H(\theta) - (1-\gamma)(F(\tilde{\theta}_f) - F(\tilde{\theta}_n))\alpha H(\theta)] = c \tag{5}$$

Finally, the threshold $\tilde{\theta}_n$ solves

$$\begin{aligned}
&\int_{\tilde{\theta}_n}^{\tilde{\theta}_f} u(q_n(\theta))f(\tilde{\theta}_n)d\theta + \int_{\tilde{\theta}_f}^{\bar{\theta}} f(\tilde{\theta}_n)\alpha u(q_f(\theta))(1-F(\theta))d\theta \\
&- \left(\theta u(q_n(\theta)) + u(q_n(\theta))\frac{F(\theta)-F(\tilde{\theta}_n)}{f(\theta)} - cq_n(\theta)\right) f(\theta) = 0
\end{aligned} \tag{6}$$

This finishes the proof of Lemma 31. ■

Therefore, there is pooling in the upper section of the demand type space, where the monopoly chooses to sell to each fair and neutral types. Thus, when both types are served, there is pooling, which is parallel to the result in the no exclusion case. But, in the intermediate range, the quantities differ for fair and neutral types, as for this range the monopoly excludes the fair types.

Lemma 32 *$q(\theta_1) \geq q(\theta_2)$ for $\theta_1 \in [\tilde{\theta}_n, \tilde{\theta}_f]$ and $\theta_2 \in [\tilde{\theta}_f, \bar{\theta}]$.*

Proof. In Equation 4 and 5 above, we have

$$[\theta + \frac{F(\theta) - F(\tilde{\theta}_n)}{f(\theta)}] \geq \theta \geq [\theta - H(\theta) - \alpha(1 - F(\tilde{\theta}_f))H(\theta) - (1-\gamma)(F(\tilde{\theta}_f) - F(\tilde{\theta}_n))\alpha H(\theta)]$$

Since $u'(\cdot)$ is a decreasing function, $q(\theta)$ should increase to hold the equation. ■

Lemma 33 *$\tilde{\theta}_f = \tilde{\theta}_n$*

Proof. By Lemma 25 we showed $\tilde{\theta}_f \geq \tilde{\theta}_n$. Suppose $\tilde{\theta}_f > \tilde{\theta}_n$. For neutral types with $\theta_1 \in [\tilde{\theta}_n, \tilde{\theta}_f]$ and $\theta_2 \in [\tilde{\theta}_f, \bar{\theta}]$ IC constraints are as follows:

$$IC_1: \quad \theta_1 q(\theta_1) - T(q(\theta_1)) \geq \theta_1 q(\theta_2) - T(q(\theta_2)) \quad \Rightarrow \quad T(q(\theta_2)) \geq T(q(\theta_1)) + \theta_1(q(\theta_2) - q(\theta_1))$$

$$IC_2: \quad \theta_2 q(\theta_2) - T(q(\theta_2)) \geq \theta_2 q(\theta_1) - T(q(\theta_1)) \quad \Rightarrow \quad T(q(\theta_1)) + \theta_2(q(\theta_2) - q(\theta_1)) \geq T(q(\theta_2))$$

Combining these two inequalities, we get,

$$T(q(\theta_1)) + \theta_2(q(\theta_2) - q(\theta_1)) \geq T(q(\theta_2)) \geq T(q(\theta_1)) + \theta_1(q(\theta_2) - q(\theta_1))$$

$$\theta_2(q(\theta_2) - q(\theta_1)) \geq \theta_1(q(\theta_2) - q(\theta_1))$$

By Lemma 32, $q(\theta_2) < q(\theta_1)$. Hence it contradicts with the supposition $\theta_2 > \theta_1$. ■

Lemma 34 *$\tilde{\theta}_f = \tilde{\theta}_n = \underline{\theta}$*

Proof. Suppose $\tilde{\theta}_f = \tilde{\theta}_n > \underline{\theta}$. Using IR constraint for $\tilde{\theta}_f$, we get

$$V(q(\tilde{\theta}_f), \tilde{\theta}_f) - \alpha[\int_{\tilde{\theta}_f}^{\bar{\theta}} (V(q(\hat{\theta}), \hat{\theta}) - V(q(\tilde{\theta}_f), \tilde{\theta}_f)) f(\hat{\theta}) d\hat{\theta}] \geq 0$$

Since $V(\cdot)$ is a strictly increasing function of θ, we have

$$V(q(\tilde{\theta}_f), \tilde{\theta}_f) > 0$$

Using linearity of $V(\cdot)$ on θ, there exist $\epsilon > 0$ such that for all $\theta \in (\tilde{\theta}_f - \epsilon, \tilde{\theta}_f)$:

$$V(q(\tilde{\theta}_f), \theta) = \theta q(\tilde{\theta}_f) - T(q(\tilde{\theta}_f)) > 0$$

Therefore, for those neutral type buyers mimicking the threshold type is better than being excluded. It contradicts the IC constraints of consumers with $\theta \in (\tilde{\theta}_f - \epsilon, \tilde{\theta}_f)$. ■

Therefore, we have shown that even if the monopoly is allowed to exclude some of the buyer types, excluding some types is just not incentive feasible. Thus, the optimal thing to do for the monopoly is to not exclude any types, as $\tilde{\theta}_f = \tilde{\theta}_n = \underline{\theta}$ suggests. Thus, our analysis in Section 4.1 provides the optimal nonlinear pricing even when we allow exclusion.