EC 203 - INTERMEDIATE MICROECONOMICS
Boğaziçi University - Department of Economics
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Problem Set 8 - Solutions

1. Each firm in a competitive market has the same cost function of $c(q)$. The market demand function is $Q_d = 24 - p$. Determine the long-run equilibrium price, quantity per firm, market quantity and number of firms when

   (a) $c(q) = 16 + q^2$

   Solution: In the long-run equilibrium price p^*, the profits are zero, that is $p^* = MC(q) = ATC(q)$. We have $MC(q) = 2q$ and $ATC(q) = 16/q + q$. Then MC=ATC implies $2q = 16/q + q$, that is, $q = 16/q$ which implies $q^* = 4$, the long-run equilibrium output per firm.

   The long-run equilibrium price is simply $MC(q^*) = 2q^* = 2 \cdot 4 = 8$.

   The market quantity is determined through the market demand, $Q_d(p^*) = 24 - p^* = 24 - 8 = 16$.

   The number of firms in the long-run $n^* = 16/4 = 4$.

   (b) $c(q) = q - q^2 + q^3$

   Solution: In the long-run equilibrium price p^*, the profits are zero, that is $p^* = MC(q) = ATC(q)$. We have $MC(q) = 1 - 2q + 3q^2$ and $ATC(q) = 1 - q + q^2$. Then $MC = ATC$ implies $1 - 2q + 3q^2 = 1 - q + q^2$, that is, $2q^2 = q$ which implies $q^* = 0.5$, the long-run equilibrium output per firm.

   The long-run equilibrium price is simply $MC(q^*) = 1 - 2q^* + 3{q^*}^2 = 1/4 = .75$.

   The market quantity is determined through the market demand, $Q_d(p^*) = 24 - p^* = 24 - .75 = 23.25$.

   The number of firms in the long-run $n^* = 23.25/.5 = 46.5$, that is, $n^* = 46$.

2. The bolt-making industry currently consists of 20 producers, all of whom operate with the identical short-run total cost curves $c(q) = 10 + q^2$ where q is the output of a firm. The market demand for bolts is $Q_d = 110 - p$ and the industry is perfectly competitive.

   (a) What is the short-run supply curve of a firm?

   Solution: First let's find the minimum of AVC curve. Find the quantity level that gives $MC = AVC$, where $MC = 2q$ and $AVC = q^2/q = q$. Then $MC(q) = AVC(q)$, implies $2q = q$ that is $q = 0$. Thus, for any $q > 0$, MC is above AVC. Thus the short-run supply curve is the entire MC curve: $p = MC(q)$, that is, $p = 2q$, or $q(p) = p/2$ is the short-run supply curve.

   (b) What is the short-run market supply curve?

   Solution: In the short-run the number of firms is fixed at 20. Thus, the short-run supply curve is the summation of these 20 firms' short-run supply curves: $Q_s(p) = 20(p/2) = 10p$.

   (c) Determine the short-run equilibrium price and quantity in this industry.

   Solution: The short-run equilibrium price is given by the equality of market supply and market demand. $Q_d(p) = 110 - p$ and $Q_s(p) = 10p$, that is, $110 - p = 10p$, which implies $11p = 110$ and $p^* = 10$. Then, the market equilibrium quantity is $Q^* = 100$.

3. Suppose that in a perfectly competitive market each firm has a long-run supply function $c(q) = kq^2$, where $k > 0$. Suppose also that the prevailing market price is $p > 0$. For what values of k and p does this firm exit the market?

   Solution: The decision to exit the market is given by $p < ATC(q)$, that is, $p < kq$. Note that firm's quantity in the long-run is given by $MC(q) = p$, which is $2kq = p$, that is, $q = p/2k$. Thus, $p < ATC(q)$ can be written as $p < k(p/2k)$, that is, $p < p/2$, which is never the case as long as $p > 0$. Thus, for any values of k and p, the firm never exits.

4. Suppose that in a perfectly competitive market each firm has a long-run marginal cost given by $MC(q) = 100 - 20q + 3q^2$, and a long-run average cost $ATC(q) = 100 - 10q + q^2$. The market demand is given by $Q_d = 22500 - 100p$.

   (a) What is the long-run competitive equilibrium price in this market?

   Solution: In the long-run, $p = MC(q) = ATC(q)$, which implies $100 - 20q + 3q^2 = 100 - 10q + q^2$, that is $2q^2 = 10q$, or $q = 5$. The price is $p^* = MC(5) = 100 - 20 \cdot 5 + 3 \cdot 5^2 = 75$.

   (b) How many firms are in this market in a long-run competitive equilibrium?

   Solution: The market equilibrium quantity is given by $Q_d = 22500 - 100 \cdot 75 = 15000$. Then the number of firms will be $n^* = 15000/5 = 3000$.

5. An industry has the demand curve $Q_d(p) = A - p$. Each of a very large number of potential firms has the long-run cost function

$$c(q) = \begin{cases} q + q^2 + 9 & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

   (a) For $A = 28$, in the long-run competitive equilibrium, find the price, the market output, output per operating firm, and number of operating firms.

   Solution: We know that in the long run price must equal the minimum average total cost of the individual firm, and

$$ATC(q) = \frac{TC(q)}{q} = 1 + q + \frac{9}{q}$$

   We know that ATC reaches its minimum where it intersects with MC, which is given by

$$MC(q) = dTC/dq = 1 + 2q$$

   and so we can solve for the value of q^* at which the minimum occurs, which is also the output per operating firm:

$$1 + q^* + \frac{9}{q^*} = 1 + 2q^* \iff q^* = 3$$

   and so the price (minimum average cost) is given by

$$p^* = ATC(3) = 1 + 3 + \frac{9}{3} = 7$$

   Finally, we have that the demand function is given by $Q_d(p) = 28 - p$. The market equilibrium output is given by

$$Q^* = Q_d(p^*) = 28 - p^* = 28 - 7 = 21$$

Then, the equilibrium number of firms is given by

$$n^* = Q^*/q^* = 21/3 = 7$$

(b) Now the demand curve shifts up in the sense that A increases to 67. In the short run, the number of firms is fixed at the number you found in part (a) above. Find the new short-run equilibrium price and per-firm output.

Solution: The number of firms is fixed at $n = 7$. From individual firms' profit maximization we know that the equilibrium price must equal marginal cost, and so we have that

$$p^* = 1 + 2q^*$$

where q^* is again the individual firm's optimal output. Equating supply and demand yields

$$Q_s(p^*) = Q_d(p^*) \Longleftrightarrow 7q^* = 67 - p^*$$

Solving these two together yields

$$p^* = \frac{47}{3} \quad \text{and} \quad q^* = \frac{22}{3}$$

(c) Now find the new long-run equilibrium for $A = 67$.

Solution: We again have, by the exact same reasoning as in part (a), that $q^* = 3^*$ and $p^* = 7$. To find n, we simply equate supply and demand as in part (a), using the new demand curve:

$$\begin{aligned} Q_s(p^*) = Q_d(p^*) &\Longleftrightarrow n \cdot q^* = 67 - p^* \\ &\Longleftrightarrow n \cdot 3 = 67 - 7 \\ &\Longleftrightarrow n = 20. \end{aligned}$$

6. The cost function of a typical firm in a competitive industry is given by $c\,(q) = 3q^3 + q$, while demand is given by $D\,(p) = 10 - p$.

(a) Suppose there are currently n such firms in the industry. What is the short-run industry supply function?

Solution: Because n is fixed in the short-run, the industry supply function is given by

$$\begin{aligned} S\,(p) &= \sum_{i=1}^{n} q_i^*\,(p) \\ &= nq^*(p), \end{aligned}$$

where $q^*(p)$ is a typical's individual firm supply function. This is the value of q which maximizes $r\,(q) - c\,(q) = pq - \left(3q^3 + q\right)$. The FOC yields

$$p - 9q^2 - 1 = 0,$$

Note that the second derivative $-18q < 0$, so the value satisfying the FOC is $q = \frac{\sqrt{p-1}}{3}$ is the solution. Observe that

$$\begin{aligned} AVC(q) &= \frac{VC(q)}{q} \\ &= \frac{3q^3 + q}{q} \\ &= 3q^2 + 1 \\ &= 3\left(\frac{\sqrt{p-1}}{3}\right)^2 + 1 \\ &= \frac{p-1}{3} + 1 \\ &= \frac{p+2}{3}, \end{aligned}$$

and so the firm's supply function is given by

$$\begin{aligned} q^*(p) &= \begin{cases} q & \text{if } p \geq AVC(q) \\ 0 & \text{if } p < AVC(q), \end{cases} \\ &= \begin{cases} \frac{\sqrt{p-1}}{3} & \text{if } p \geq \frac{p+2}{3} \\ 0 & \text{if } p < \frac{p+2}{3}, \end{cases} \\ &= \begin{cases} \frac{\sqrt{p-1}}{3} & \text{if } p \geq 1 \\ 0 & \text{if } p < 1, \end{cases} \end{aligned}$$

and so

$$S(p) = nq^*(p) = \begin{cases} \frac{n\sqrt{p-1}}{3} & \text{if } p \geq 1 \\ 0 & \text{if } p < 1. \end{cases}$$

(b) What is the long-run industry supply function?

Solution: The long-run industry cost function is determined by profit maximization ($p = c'(q^*(p))$) and zero profit (free entry) ($p = ATC(q^*(p))$), and so $c'(q^*(p)) = ATC(q^*(p))$, which we know only occurs when

$$\begin{aligned} p &= p^{\min} \equiv \min\ ATC(q) \\ &= \min\left\{\frac{c(q)}{q}\right\} \\ &= \min\left\{\frac{3q^3 + q}{q}\right\} \\ &= \min\left\{3q^2 + 1\right\} \\ &= 1. \quad (\text{since } q \geq 0 \text{ must hold}), \end{aligned}$$

and so the long-run industry supply curve has slope zero (perfectly elastic) and runs through all quantities at $p = 1$.

(c) What is the short-run industry equilibrium, (p, q, Q)?

Solution: The three equations characterizing industry equilibrium in the short run are

$$
\begin{aligned}
&nq = D(p) && \text{(supply = demand)} \\
&p = c'(q) && \text{(profit maximization)} \\
&Q = nq && \text{(market quantity = sum of individual quantities),}
\end{aligned}
$$

which here give us

$$
\begin{aligned}
nq &= 10 - p, \\
p &= 9q^2 + 1, \\
Q &= nq.
\end{aligned}
$$

Substituting the second equation into the first yields

$$
\begin{aligned}
nq = 10 - 9q^2 - 1 &\iff 9q^2 + nq - 9 = 0 \\
&\iff q^2 + \left(\frac{n}{9}\right) q - 1 = 0 \\
&\iff q = \frac{\sqrt{n^2 + 324} - n}{18},
\end{aligned}
$$

which plugged back into the first equation yields

$$
\begin{aligned}
p &= 10 - nq \\
&= 10 - \frac{n\left(\sqrt{n^2 + 324} - n\right)}{18},
\end{aligned}
$$

(which can be confirmed is never less than 1 for any positive n). Finally we have

$$
\begin{aligned}
Q &= nq \\
&= \frac{n\left(\sqrt{n^2 + 324} - n\right)}{18}.
\end{aligned}
$$

(d) What is the profit of a typical firm in the short-run?

Solution: The profit for a firm in the short-run is given by

$$
\begin{aligned}
r(q) - c(q) &= pq - \left(3q^3 + q\right) = q\left(p - 3q^2 - 1\right) \\
&= \frac{\sqrt{n^2 + 324} - n}{18}\left(10 - \frac{n\left(\sqrt{n^2 + 324} - n\right)}{18} - 3\left(\frac{\sqrt{n^2 + 324} - n}{18}\right)^2 - 1\right) \\
&= \frac{1}{216} n^2 \sqrt{n^2 + 324} - n + \frac{1}{2}\sqrt{n^2 + 324} - \frac{1}{1944}\left(n^2 + 324\right)^{\frac{3}{2}} - \frac{1}{243} n^3
\end{aligned}
$$

This profit approaches to zero as n goes to infinity.

(e) What is the long-run industry equilibrium, (p, q, Q, n)?

Solution: The four equations characterizing industry equilibrium in the short run are

$$\begin{aligned} nq &= D(p) && \text{(supply = demand)} \\ p &= c'(q) && \text{(profit maximization)} \\ Q &= nq && \text{(market quantity = sum of individual quantities),} \\ p &= ATC(q) && \text{(zero profit)} \end{aligned}$$

can be solved to yield

$$\begin{aligned} p &= p^{\min} \\ q &= q^{\min} \\ Q &= D\left(p^{\min}\right), \\ n &= \frac{D\left(p^{\min}\right)}{q^{\min}} \end{aligned}$$

where $q^{\min}$ is defined as the output level which satisfies the condition $p^{\min} = ATC\left(q^{\min}\right)$. Because $ATC(q) = 3q^2 + 1$, we have that $p^{\min} = 1$ and $q^{\min} = 0$, and so the long-run industry equilibrium is

$$\begin{aligned} p &= 1, \\ q &= 0, \\ Q &= 9, \\ n &= \infty. \end{aligned}$$

That is, it will take an infinite(!) number of firms to drive profits down to zero, and when there are an infinite number of firms they will each produce "zero". The total quantity produced will be equal to $Q = 9$ in order to meet the demand at price $p = 1$, but because these 9 units are divided by infinity, the amount per firm becomes zero.

7. Suppose a monopoly's inverse demand curve is $p = 13 - Q$ and its cost function is $C(Q) = 25 + Q + Q^2/2$. What is the profit maximizing quantity and price if there is no price discrimination? What is the maximum profit? Should the monopoly operate or shut-down?

Solution: Profit is given by $\Pi = p(Q) \cdot Q - [25 + Q + Q^2/2] = (13 - Q)Q - 25 - Q - Q^2/2 = 13Q - Q^2 - 25 - Q - Q^2/2 = 12Q - 3Q^2/2 - 25$. The profit maximizing quantity satisfies the first order condition: $d\pi/dQ = 0$, that is, $0 = 12 - 3Q$, that is, $Q^m = 4$. Then the monopoly price is given by $p^m = 13 - Q^m = 13 - 4 = 9$.

The profit is then $\Pi = 12Q - 3Q^2/2 - 25 = 12 \cdot 4 - 34^2/2 = 48 - 24 - 25 = -1$

Shutting down means producing 0 and paying the fixed cost, that is $\Pi_{sd} = -25$.

Thus, the monopoly does not shut down.

8. Suppose that a monopoly faces a market demand given by $p = 30 - 2Q$. Its total cost is $C(Q) = 5 + Q^2$. Suppose the monopoly sets a uniform price. Show that the monopoly operates on the elastic portion of the demand curve. What is the profit level?

Solution: The profit is $\Pi(Q) = p(Q)Q - 5 - Q^2 = (30 - 2Q)Q - 5 - Q^2$. Then, the first order condition is $\Pi(Q) = 30 - 6Q = 0$ which implies $Q^M = 5$, $p^M = 20$. At these quantity and price levels, the elasticity is given by $\epsilon = |(dQ/dp)(p/Q) = -(-1/2)(20/5)| = 2 > 1$, thus it is on the elastic portion of the demand curve. The profit is $\Pi^M = 70$.

9. A monopoly has cost function $C(Q) = 6Q$. The output Q is consumed only by consumer a and consumer b. Consumer a has the demand function $Q_a(p) = 10 - p$, and that of consumer b is $Q_b(p) = 20 - p$.

   (a) Find the industry demand function $Q(p)$; the inverse demand function $p(Q)$; the revenue function $R(Q)$; and the marginal revenue function $MR(Q)$.

   Solution We find industry demand by simply adding the individual demand curves, so

$$
\begin{aligned}
Q(p) &= \begin{cases} Q_a(p) + Q_b(p) & \text{if } p \leq 10 \\ Q_b(p) & \text{if } 10 < p \leq 20 \\ 0 & \text{if } p > 20 \end{cases} \\
&= \begin{cases} 30 - 2p & \text{if } p \leq 10 \\ 20 - p & \text{if } 10 < p \leq 20 \\ 0 & \text{if } p > 20. \end{cases}
\end{aligned}
$$

   The inverse demand is then given by

$$
p(Q) = \begin{cases} 20 - Q & \text{if } Q \leq 10 \\ \frac{30-Q}{2} & \text{if } 10 < Q \leq 30 \\ 0 & \text{if } Q > 30. \end{cases}
$$

   The revenue function is

$$
\begin{aligned}
R(Q) &= Q\,p(Q) \\
&= Q\,p(Q) \\
&= \begin{cases} 20Q - Q^2 & \text{if } Q \leq 10 \\ \frac{30Q-Q^2}{2} & \text{if } 10 < Q \leq 30 \\ 0 & \text{if } Q > 30, \end{cases}
\end{aligned}
$$

and so marginal revenue is

$$MR(Q) = \begin{cases} 20 - 2Q & \text{if } Q \leq 10 \\ 15 - Q & \text{if } 10 < Q \leq 30 \\ 0 & \text{if } Q > 30. \end{cases}$$

(b) Find the monopoly output Q^M and price p^M.

Solution By equating marginal revenue with marginal cost we get

$$MR(Q) = MC(Q) = 6$$

Note that for $10 < Q \leq 30$, this means $15 - Q = 6 \Longleftrightarrow Q = 9$, but this contradicts $10 < Q \leq 30$ and so we have the solution for when $Q \leq 10$, which is

$$20 - 2Q^M = 6 \Longleftrightarrow Q^M = 7.$$

The price is then simply given by

$$\begin{aligned} p^M &= p(Q^M) \\ &= 20 - 7 \\ &= 13. \end{aligned}$$