Algebra II

Homework 4 - Due April 25, 2024

1. Let F be a field, and T an indeterminate. Let $f, g \in F[T]$ be relatively prime. Suppose also that it is not the case that both f and g constant and that $g \neq 0$. Show that
$$[F(T) : F(\frac{f}{g})] = \max\{\deg f, \deg g\}.$$

2. Let $\alpha = \sqrt{2+\sqrt{2}}$, and $K = \mathbb{Q}(\alpha, i)$.
   (a) Show that $K|\mathbb{Q}$ is Galois and find its Galois group in terms of elementary groups.
   (b) Find a primitive element of K over $\mathbb{Q}$. Justify your answer.

3. Let $K = \mathbb{C}(T)$ where T is an indeterminate, and let $\zeta \in \mathbb{C}$ be a primitive third root of unity. Also let σ, τ be the automorphisms of K over $\mathbb{C}$ with $\sigma(T) = \zeta T$ and $\tau(T) = T^{-1}$. Find $f \in K$ such that the fixed field of the group of automorphisms of K generated by σ and τ is $\mathbb{C}(f)$.